Probing Neutrino Properties with Astrophysics: Neutrino Masses

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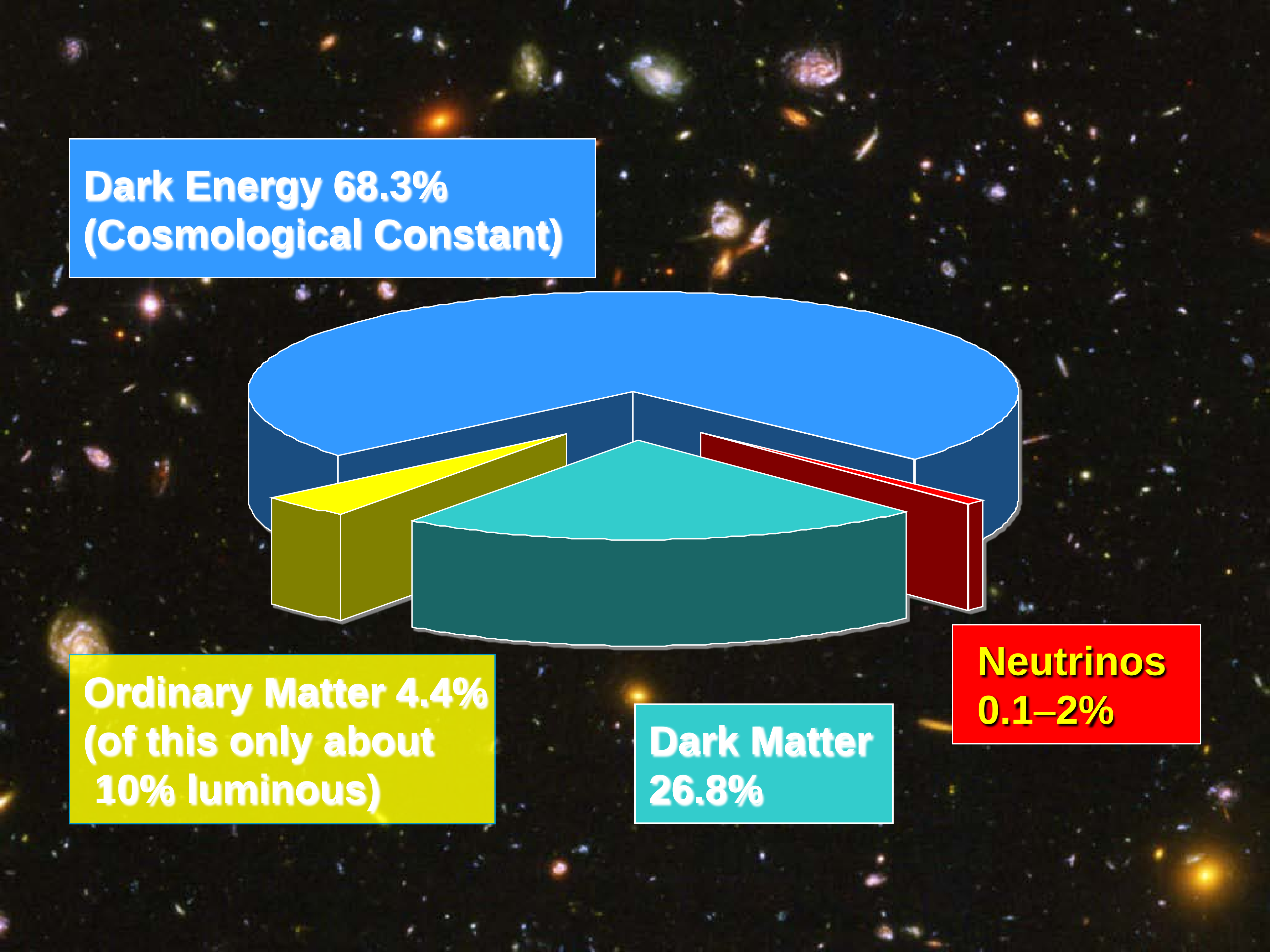
International Workshop on UNICOS 2014
13-15 May, 2014
Panjab University, Chandigarh, India


Dark Energy 68.3%
(Cosmological Constant)
Ordinary Matter 4.4%
(of this only about
10% luminous)
Dark Matter
26.8%
Neutrinos
0.1–2%

Contents:

- Theoretical Aspects of Neutrino Masses and Lepton Mixing Angles:
- Absolute Neutrino Masses from laboratory tests. (S. Goswami's talk)
- Neutrino Masses in cosmology (CMB, Power Spectrum in Large Scale Structures: Lambda CDM vs INuDE-Model
- Discussions

Papers:

X.G. He, YYK, and RR Volkas, JHEP 0604 (2006) 039.

YYK and K. Ichiki, JCAP 0806, 005, 2008;

JHEP 0806, 058, 2008;

arXiv:0803.3142

Theoretical Aspects:

A Non-vanishing neutrino mass is the first evidence of the incompleteness of the Standard Model[SM].

Questions:

- How to extend the SM in order to accommodate neutrino masses ?
- Why neutrino masses are so small, compared with the charged fermion masses?
- Why lepton mixing angles are so different from those of the quark sector?

How to modify the SM ?

The SM, as a consistent QFT, is completely specified by

0. Invariance under local transformations of the gauge group $G = SU(3) \times SU(2) \times U(1)_Y$ [plus Lorentz invariance]
1. Particle content: three copies of (Q, u^c, d^c, L, e^c)
   one Higgs doublet ϕ
2. Renormalizablity (i.e. the requirement that all coupling constant g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

We can not give up gauge invariance !. It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory.

We could extend G, but to allow for neutrino masses, we need to modify 1, (and/or 2) anyway ...

First Possibility: modify (1), the particle content

There are several possiblities:
One of the simplest one is to mimic the charged fermion sector:

- add (three copies of) right-handed neutrinos $\nu^c = (1, 1, 0)$ full-singlet under $G = SU3) \times SU(2) \times U(1)$
- ask for (global) invariance under $B - L$ (no more automatically conserved as in the SM)

The neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = d^c y_d(\phi^+ Q) + u^c y_u(\tilde{\phi}^+ Q) + e^c y_e(\phi^+ L) + \nu^c y_\nu(\tilde{\phi}^+ L) + h.c. \tag{13}$$

$$m_f = \frac{y_f}{\sqrt{2}} v, \qquad (f = u, d, e, \nu) \tag{14}$$

With three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \, \bar{e}\, \sigma^\mu \, U_{PMNS} \nu + h.c. \tag{15}$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}.

A Generic Problem of this Approach

The particle content can be modified in sevaral different ways in order to account for non-vanishing neutrino masses (additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet, SUSY particles,....)

Which is the correct one ?

A problem of the above example:
If neutrinos are so similar to the other fermions, why are so light ?

$$\frac{y_\nu}{y_{top}} \leq 10^{-12} \tag{16}$$

Second Possiblity: abandon (2) renormalizability

A disaster ?

$$L = L^{SM}_{d\leq 4} + \frac{L_5}{\Lambda} + \frac{L_6}{\Lambda^2} + ... \tag{17}$$

A new scale Λ eneters the theory. The new (gauge invariant !) operators $L_5, L_6, ...$ contribute to amplitudes for physical processes with terms of the type:

$$\frac{L_5}{\Lambda} \rightarrow \frac{E}{\Lambda}, \qquad \frac{L_6}{\Lambda^2} \rightarrow \left(\frac{E}{\Lambda}\right)^2, ... \tag{18}$$

The theory cannot be extrapolated beyond a certain energy scale $E \sim \lambda$. [at variance with a renormalizable(asymptotically free) QFT]

If $E << \Lambda$ (for example E close to the electroweak scale, 10^2 GeV, and $\Lambda \simeq 10^{15}$ GeV not far from the so-called Grand Unified scale), the above effects will be tiny and, the theory will look like a renormalizable theory !

$$\frac{E}{\Lambda} \simeq \frac{10^2\ GeV}{10^{15}\ GeV} = 10^{-13} \tag{19}$$

$\Rightarrow$ an extremely tiny effect, but exactly what needed to suppress m_ν compared to m_{top} !

Continue

Worth to explore: The dominant operators (suppressed by a single power $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators:

$$\frac{L_5}{\Lambda} = \frac{(\tilde{\phi}^{+}L)(\tilde{\phi}^{+}L)}{\Lambda} = \frac{v}{\sqrt{2}}\left(\frac{v}{\Lambda}\right)\nu\nu + ... \tag{20}$$

- a unique operator ! [up to flavour combinations] it violates (B-L) by two units.
- it is usppressed by a factor (v/Λ) wrt the neutrino mass term of above example.

$$\nu^c\,(\tilde{\phi}^{+}L) = \frac{v}{\sqrt{2}}\nu^c\nu + ... \tag{21}$$

It provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violation, is very large.
How large ? up to about 10^{15} GeV.
From this point of view neutrinos offer a unique window on physics at very large scale, inaccessible in present (and probably future) man-made experiments.
Since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos... and indeed this was the case !

Flavor Symmetries (the Hierarchy Puzzle)

Hierarchies in fermion spectrum:

- Quarks:

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1, \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1, \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1 \tag{22}$$

- Leptons:

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1, \quad \frac{\Delta m^2_{sol}}{\Delta^2_{atm}} = (0.025 \pm 0.049) \simeq \lambda^2 << 1 \ (2\sigma) \quad |U_{e3}| < 0.18 \leq \lambda \ (2\sigma) \tag{23}$$

Call ξ_i the generic small parameter. A modern approach to understand why $\xi_i << 1$ consists in regarding ξ_i as small breaking terms of an approximate flavour symmetry. When $\xi_i = 0$ the theory becomes invariant under a flavour symmetry F.

Example: why $y_e << y_{top}$? Assume $F = U(1)_F$:

$$F(t) = F(t^c) = F(h) = 0 \qquad\qquad y_{top}(h+v)\, t^c\, t \quad (allowed) \tag{24}$$

$$F(e^c) = p > 0, F(e) = q > 0 \qquad y_e(h+v)\, e^c\, e \quad (breaks\ U(1)_F\ by\ (p+q)\ units) \tag{25}$$

If $\xi =< \phi > /\Lambda < 1$ breaks U(1) by one negative unit:

$y_e \simeq O(\xi^{p+q}) << y_{top} \simeq O(1)$ provides a qualitative picture of the existing hierarchies in the fermion spectrum.

Flavour Symmetry-lepton mixing puzzle

Why $U = PMNS = U_e^\dagger U_\nu = U_{TB}$ [TB=TriBimaximal] ?

$$U_{PMNS} = (U_L^l)^\dagger U_L^\nu \simeq \begin{pmatrix} 0.85 & 0.52 & 0.053 \\ -0.33 & 0.62 & -0.72 \\ -0.40 & 0.59 & 0.70 \end{pmatrix} \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} . \quad (26)$$

Consider a flavour symmetry G_f such that G_f is broken into two different subgroups: G_e in the charged lepton sector, and G_ν in the neutrino sector.
m_e is invariant under G_e and m_ν is invariant under G_ν.

If G_e and G_ν are appropriately chosen, the constraints on m_e and m_ν can give rise to the observed U_{PMNS}.

$$G_f \quad \rightarrow G_e \rightarrow m_e \; diagonal \quad (27)$$

$$\rightarrow G_\nu \rightarrow U_{TB}^T m_\nu U_{TB} = (m_\nu)_{diag} \quad (28)$$

The Harrison-Perkins-Scott way to get tribimaximal mixing:

$$M_l = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \qquad M_\nu = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix} \tag{35}$$

– C_3 symmetry for charged leptons
– $Z_2 \otimes Z_2$: $\nu_e \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow -\nu_\mu$ for neutrinos
– Matrices form-diagonalisable; masses arbitrary

- The trimaximal charged-lepton and the ν_e/ν_τ twofold-maximal left-diagonalisation matrices to give tribimaximal MNSP(with phases).

For charged lepton case, it is quite possible that the reason why the observed CKM matrix is nearly identity , is the hierarchical breaking

A4 → Z3 ~ C3={1,c,a} → nothing

with the small mixing angles generated by higher order effects after the relatively weak subsequent breaking of the residual C3.
In neutrino sector, the flavor breaking pattern is

A4 → Z2 ={1, r2}

The Known unknowns:

what are the potential astrophysical consequences ?

- We know two mass^2 differences, but not the absolute scale of ν-mass.
- We know three mixing angles, two large mixing angles and one small mixing angle.
- We do not know the Dirac/Majorana nature of the mass.
- We do not know the hierarchy, normal and inverted.

- We do not know the sizes or roles in nature of three CP-violating phases.
- We have not explored other MSW crossings or potentials.
- We do not know whether neutrinos have nonzero electro-magnetic moments.
- We do not know the high-energy limits of ν physics.
- We do not know whether there are additional ν species.

Neutrino Mixing Matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = UV \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 e^{-i\delta} \\ -c_1 s_3 - s_2 s_1 c_3 e^{i\delta} & c_1 s_3 - s_2 s_1 s_3 e^{i\delta} & c_2 s_1 \\ s_1 s_3 - s_2 c_1 c_3 e^{i\delta} & -s_1 c_3 - s_2 c_1 s_3 e^{i\delta} & c_2 c_1 \end{pmatrix} V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where $\mathrm{V{=}diag}(1,\, \mathrm{e}^{\mathrm{i}\phi_1},\, \mathrm{e}^{\mathrm{i}(\delta+\phi_2)})$

δ = Dirac Phase; $\phi_{1,2}$ = Majorana Phases

Present Data with 3σ ranges of mixing paramters:

$\otimes$ $\theta_2 \sim 9^0$ is small: $0.07 \le \sin^2 2\theta_2 \le 0.12 \Rightarrow$ small mixing angle

(Daya Bay/RENO/Double Choose)

$\otimes$ Solar Neutrino Data $\Rightarrow$ Large Mixing Angle Sol.

$$0.70 \le \sin^2 2\theta_3 \le 0.94$$

$$7.1 \times 10^{-5} \le \Delta\mathrm{s} \le 8.9 \times 10^{-5}\ [\mathrm{eV}^2]\,; \qquad \Delta\mathrm{s}{=}m_2^2 - m_1^2$$

$\otimes$ Atmospheric Neutrino Data $\Rightarrow$ Maximal Mixing Angle Sol.

$$\sin^2 2\theta_1 \ge 0.87$$

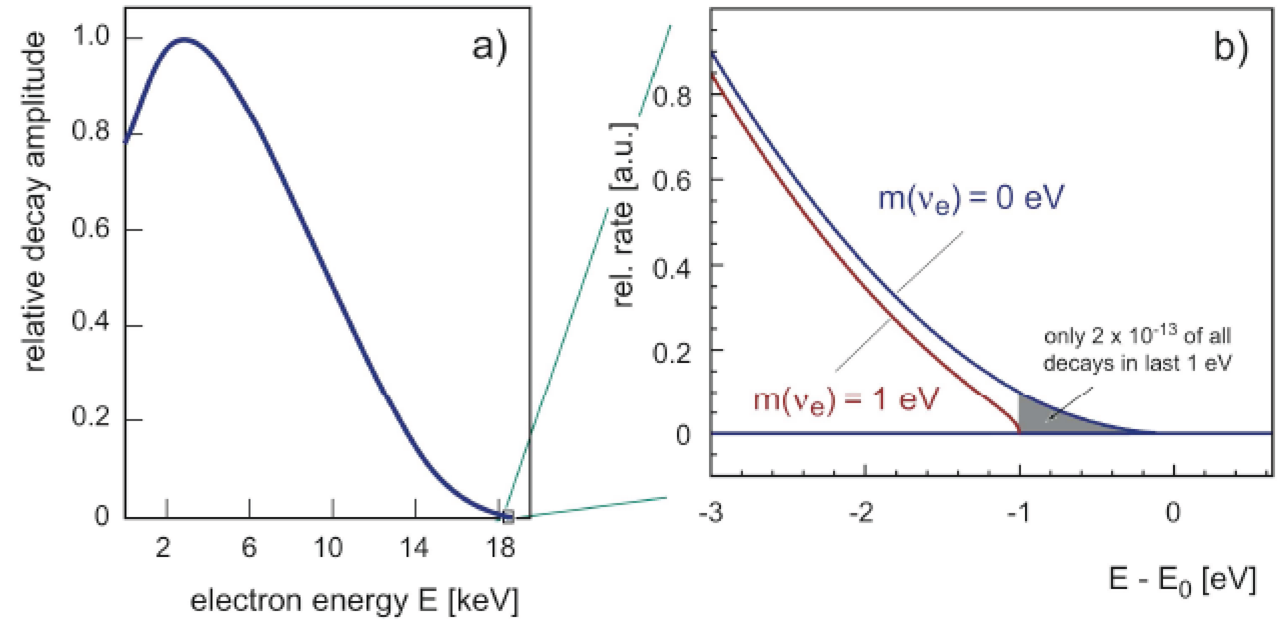
$$1.4 \times 10^{-3} \le |\Delta\mathrm{a}| \le 3.3 \times 10^{-3}\ [\mathrm{eV}^2]; \qquad \Delta\mathrm{a}{=}m_3^2 - m_1^2$$

Neutrino Masses:

three important laboratory tests

- Direct kinematic tests:

$$m^2_{\overline{\nu_e}} = \sum_{i=1}^{3} |U_{ei}|^2 \; m_i^2 \quad ; \quad \overline{\nu_e} \sim 0.8\ \nu_1 + 0.5\ \nu_2 + 0.15\ \nu_3$$



Known splittings << resolution: 2.2 eV limit $\Rightarrow \sum_{i=1}^{n} m_\nu(i) \leq 6.6$ eV

- Neutrino oscillations:

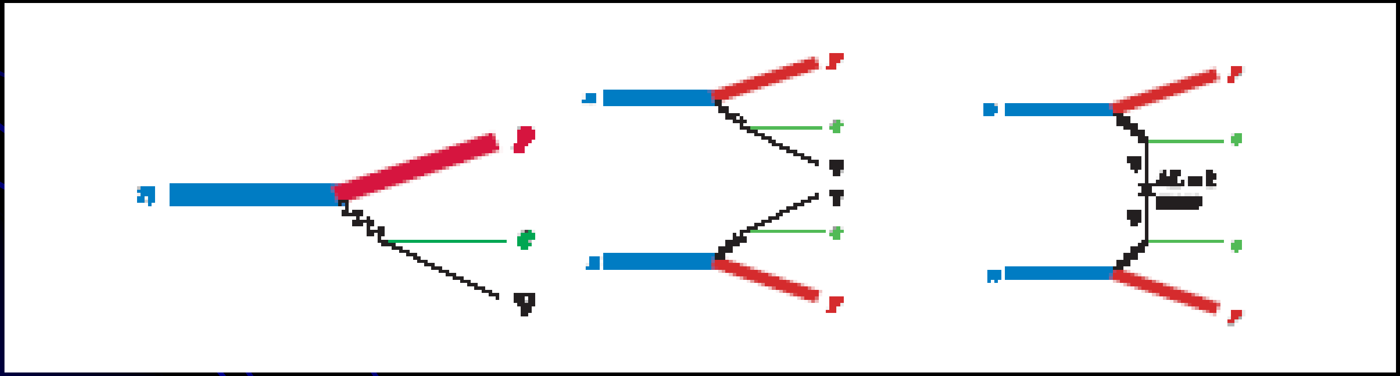
$$P_{\nu_\mu \to \nu_e} = \sin 2\theta_{12}^2 \quad \sin \frac{\triangle m_{12}^2 \ \mathrm{L}^2}{4\,E}$$

- Neutrinoless double beta decay:

$$\left\langle m_\nu^{\beta\beta} \right\rangle = \sum_{\mathrm{k=1}}^{3} \mathrm{U}_{\mathrm{ek}}^2 \ m_k$$

Absolute Neutrino Masses
in three important laboratory tests

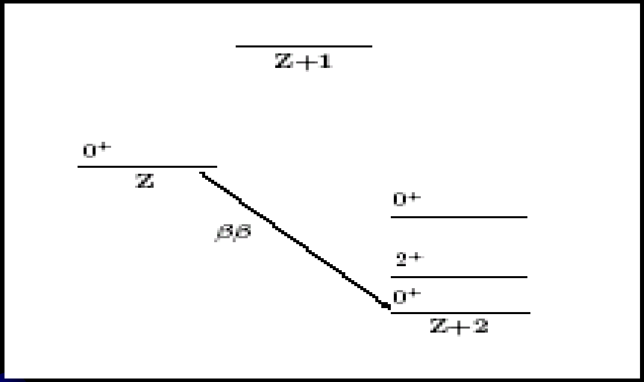
Part I

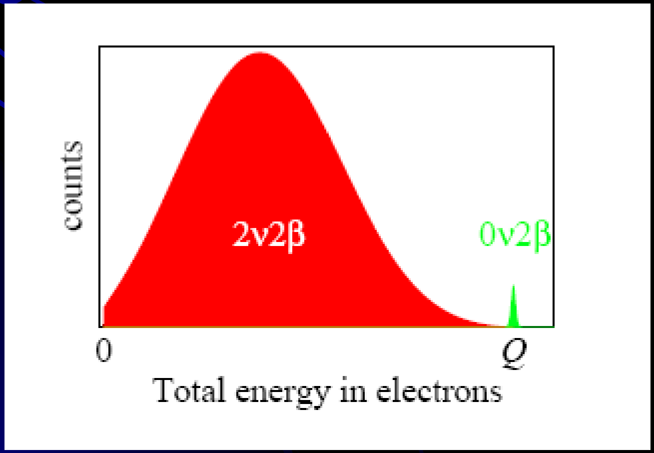


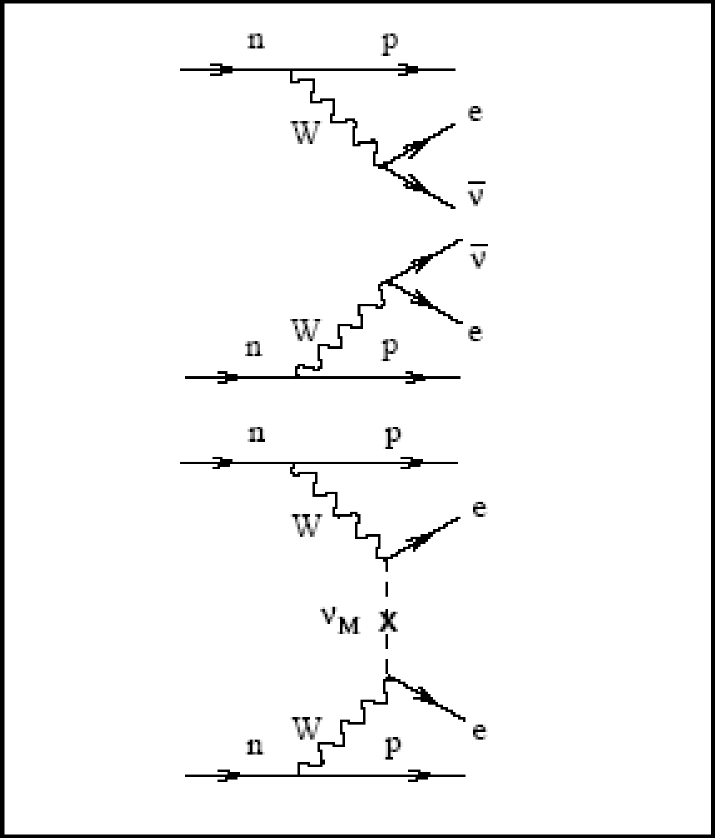
Neutrinoless double-beta decay

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$ $(\Delta L=2)$

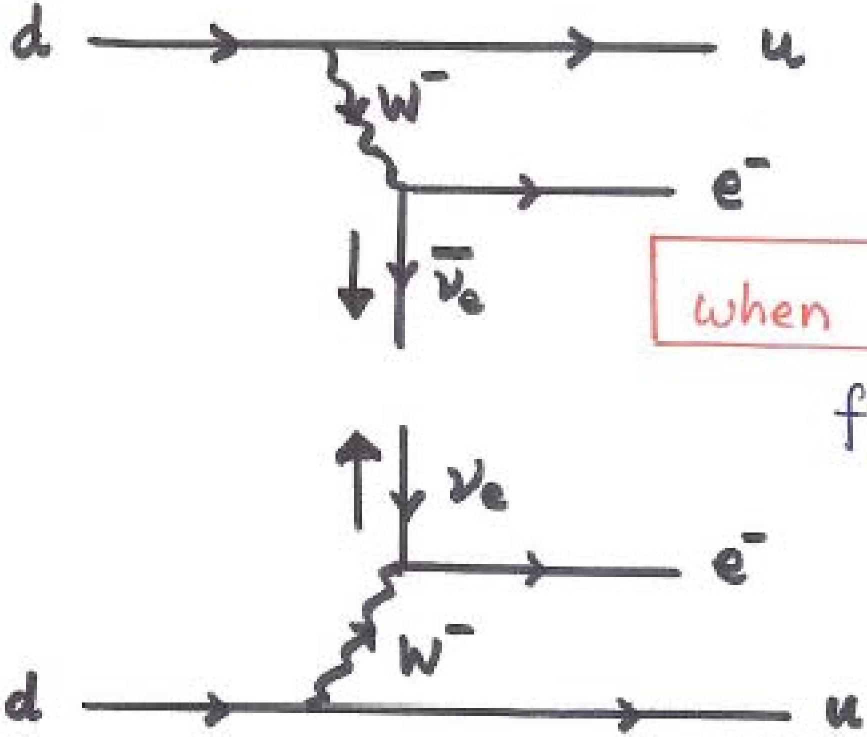
-- the most sensitive process to the total lepton number and small majorana neutrino masses





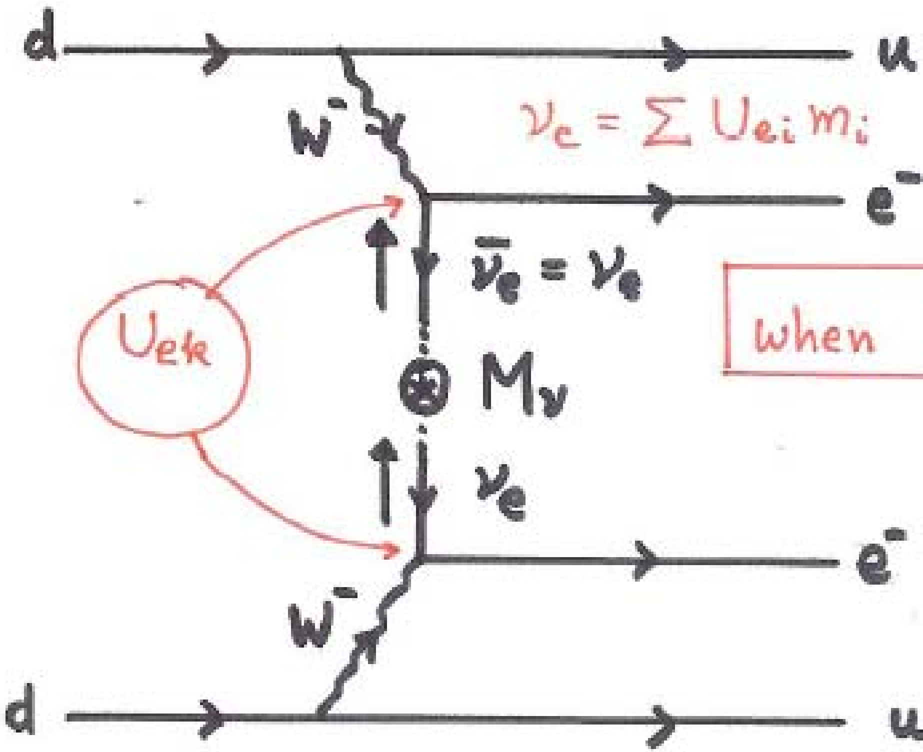


Neutrinoless Double β-decays



when $\nu_e \neq \bar{\nu}_e$, helicity mismatch

forbidden in SM.



$\boxed{\text{when } \nu_e = \bar{\nu}_e \text{, helicity matching}}$

Majorana Neutrinos

$$m_{\beta\beta} = \sum_{k=1}^{3} U_{ek}^2 \, m_k$$

$$\frac{1}{T_{1/2}^{0\nu}(A,Z)} = |m_{\beta\beta}|^2 \; |M^{0\nu}(A,Z)|^2 \; g_A^2 \; G_{01}^{0\nu}(E_0,Z)$$

where

$T_{1/2}^{0\nu}(A,Z)$ = half-life time; $M^{0\nu}$ =nuclear matrix element

$G_{01}^{0\nu}$ = phase space factor;

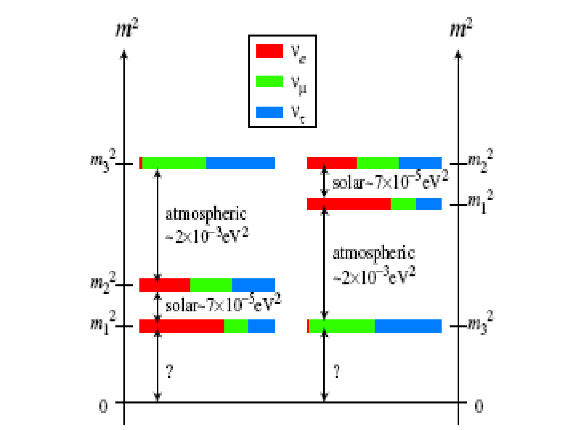
$$m_{\beta\beta} = U_{e1}^2 \; m_1 + U_{e2}^2 \; m_2 + U_{e3}^2 \; m_3$$

- 0νββ-decay has not yet been seen experimentally.
- Heidelberg-Moscow (HM) ^{76}Ge experiment:
- $T^0_{1/2} > 1.9 \times 10^{25}$ years → $|m_{\beta\beta}| < 0.55$ eV

New results from experiments using ^{136}Xe:

- $T^0_{1/2} > 1.9 \times 10^{25}$ years at 90% CL (KamLAND-ZEN:2013)
- $T^0_{1/2} > 1.6 \times 10^{25}$ years at 90% C.L. (EXO) (2012)
- $T^0_{1/2} > 3.4 \times 10^{25}$ years at 90% CL (Combined, 2013) → $|m_{\beta\beta}| < 0.25$ eV

Neutrioless Double-beta decay vs Neutrino Mass



- Mass Ordering (for simplicity)

$m_1 < m_2 < m_3$ (non-negative m_i)

Two possible mass spectra: $\Delta s = m_2^2 - m_1^2,\ \Delta a = m_3^2 - m_2^2$ (normal hierarchy);

$\Delta s = m_3^2 - m_2^2,\ \Delta a = m_2^2 - m_1^2$ (inverted hierarchy)

where $\Delta a >> \Delta s$

- The rate of $0\nu\beta\beta$ decay depends on the mag. of the element of the neutrino mass matrix: $\nu_e - \nu_e$

$$M_{ee} = |c_2^2 c_3^2 m_1 + c_2^2 s_3^2 m_2 e^{i\phi_2} + s_2^2 m_3 e^{i\phi_3}| \quad \text{(Case I);}$$

$$= |c_2^2 c_3^2 m_3 + c_2^2 s_3^2 m_2 e^{i\phi_2} + s_2^2 m_1 e^{i\phi_3}| \quad \text{(Case II);}$$

m_i may be determined from the lightest mass m and mass-squared differences

Bound of the total neutrino mass

Since $\sin^2 2\theta_2 \sim 0.1$ or $s_2^2 \sim 0.024$,

The limit on Σ for $\theta_2 = 0$ are

$$2\,\mathrm{M_{ee}} + \sqrt{M_{ee}^2 \pm \Delta} \;\le\; \Sigma \;\le\; \frac{2\,\mathrm{M_{ee}} + \sqrt{M_{ee}^2 \pm \Delta \cos^2 2\theta_3}}{|\cos 2\theta_3|}$$

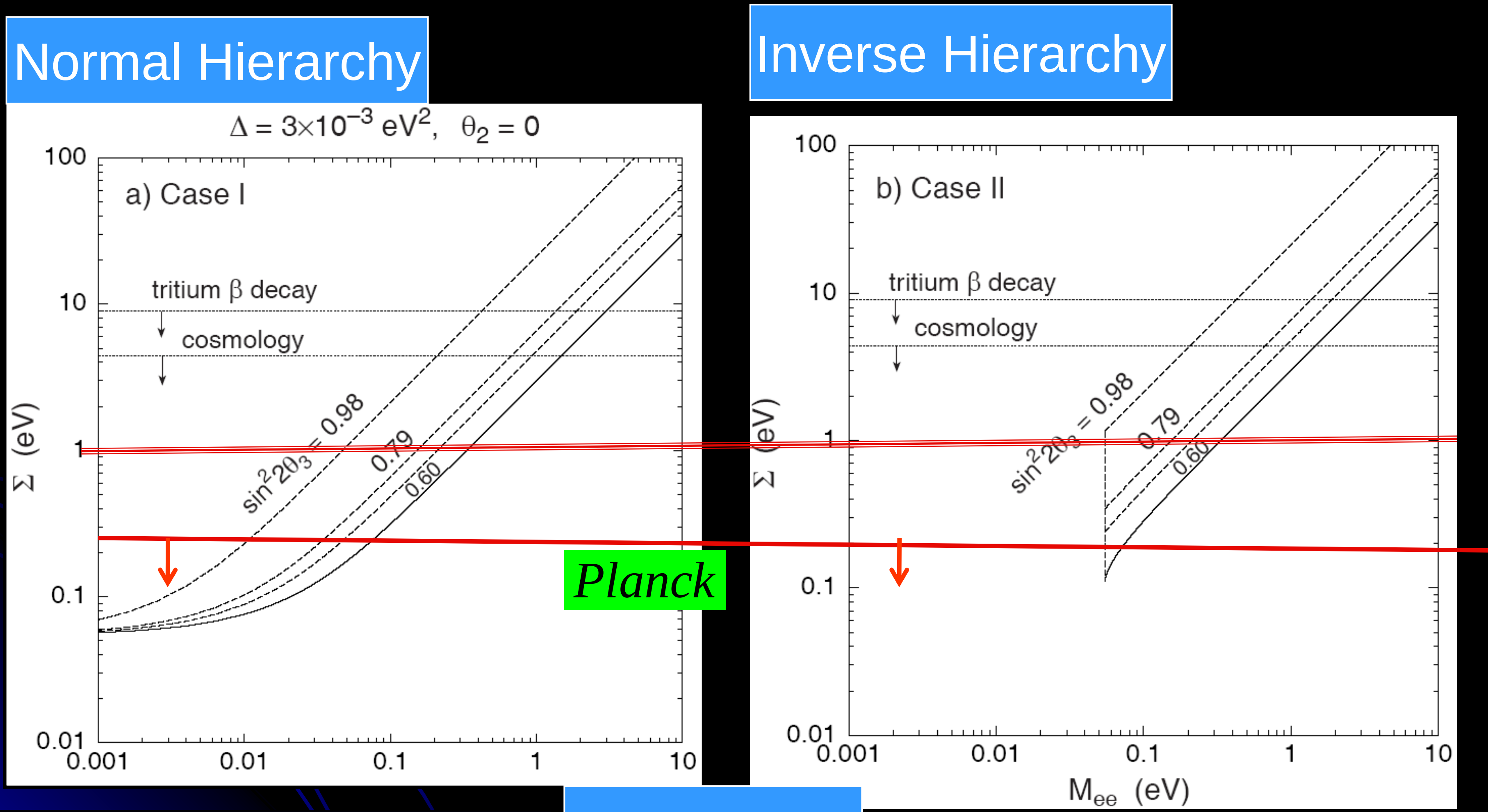
(+ for Case I and -- for Case II)

- Depends on two parameters;

(1) the scale of atm. Neutrino Osci, (Δ)

(2) the amplitude of solar Neutrino Osci. ($\sin^2 2\theta_3$)

Total Nu-Mass vs Mee (NH vs IH)



Compare with S. Sharma works in poster

Mee(eV)

Effective Majorana Nu-mass

$\Delta m^2_{sol} = 7.1 \bullet 10^{-5} eV^2,\ \Delta m^2_{atm} = 2.0 \bullet 10^{-3} eV^2,\ \sin^2\theta_{12} = 0.29$ [best fit values]

$\sin^2\theta_{13} = 0.024$

Normal hierarchy of ν masses: $m_1 \ll m_2 \ll m_3$				
m_1 [10^{-3} eV]	m_2 [10^{-3} eV]	m_3 [10^{-2} eV]	$sin^2\theta_{13}$	$\|m_{\beta\beta}\|$ [10^{-3} eV]
(0, 1.7)	(8.43, 8.60)	(4.47, 4.48)	0.00	(1.29, 3.70)
			0.01	(0.83, 4.11)
			0.05	(0.00, 5.75)

Inverted hierarchy of ν masses: $m_3 \ll m_1 < m_2$				
m_3 [10^{-3} eV]	m_1 [10^{-2} eV]	m_2 [10^{-2} eV]	$sin^2\theta_{13}$	$\|m_{\beta\beta}\|$ [10^{-2} eV]
(0, 8.9)	(4.39, 4.48)	(4.47, 4.56)	0.00	(1.82, 4.50)
			0.01	(1.80, 4.47)
			0.05	(1.72, 4.32)

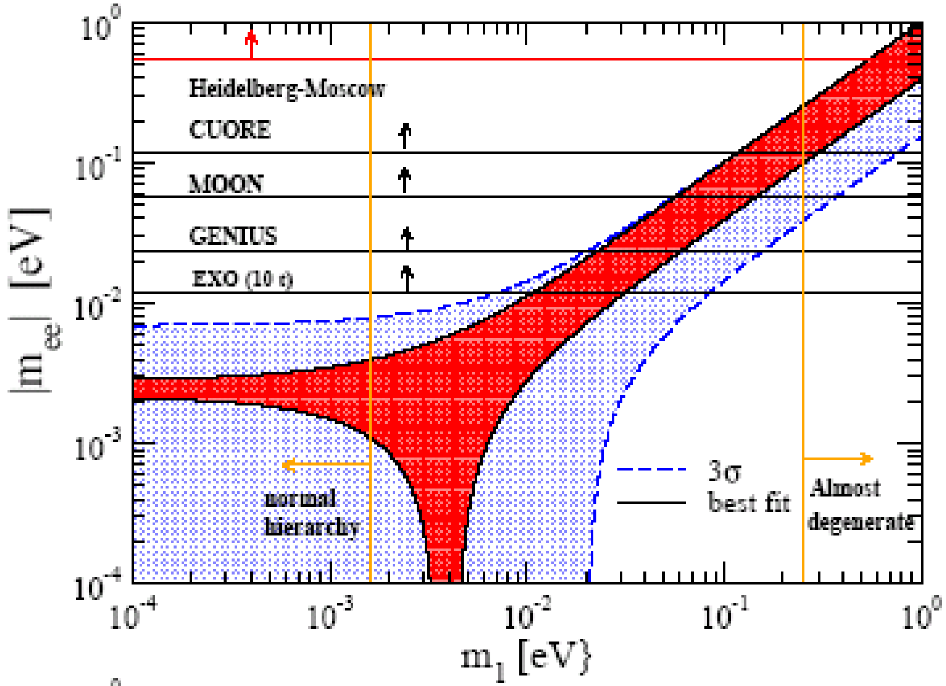
Almost degenerate ν mass spectrum: $m_1 \simeq m_2 \simeq m_3$				
m_1 [eV]	m_2 [eV]	m_3 [eV]	$sin^2\theta_{13}$	$\|m_{\beta\beta}\|$ [eV]
(0.22, 0.60)	(0.22, 0.60)	(0.22, 0.60)	0.00	(0.092, 0.60)
			0.01	(0.089, 0.60)
			0.05	(0.077, 0.60)

Sensitivities of the future exps.

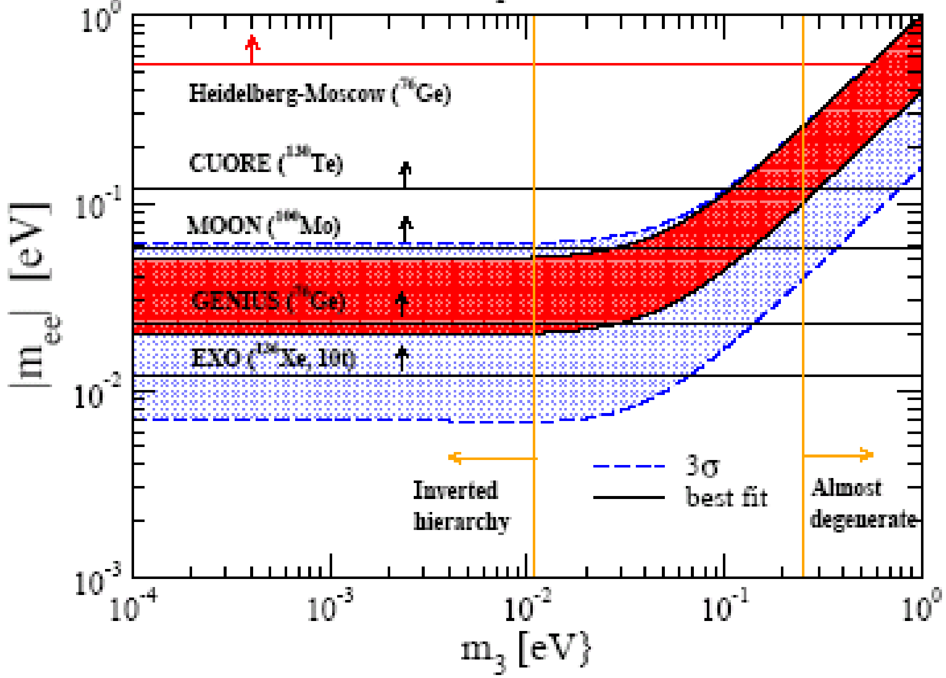
nucl.	$M^{0\nu}$	$M_{GT}^{2\nu-exp}$	$\mathcal{R}^{2\nu/0\nu}$	$T_{1/2}^{2\nu-exp}$ Ref.	$T_{1/2}^{0\nu}$ Ref.	Exp.	$\lvert m_{\beta\beta}\rvert$
		MeV^{-1}	MeV^{-1}	years	years		eV
^{76}Ge	2.40	0.15	0.063	$1.3\ 10^{21}$[20]	$1.9\ 10^{25}$[37]	HM	0.55
					$3\ 10^{27}$[20]	Majorana	$0.044^{\dagger}$
					$7\ 10^{27}$[20]	GEM	$0.028^{\dagger}$
					$1\ 10^{28}$[20]	GENIUS	$0.023^{\dagger}$
^{100}Mo	1.16	0.22	0.19	$8.0\ 10^{18}$[20]	$6.0\ 10^{22}$[74]	NEMO3	7.8
					$4\ 10^{24}$[20]	NEMO3	0.92^{*}
					$1\ 10^{27}$[20]	MOON	$0.058^{\dagger}$
^{130}Te	1.50	0.017	0.013	$6.1\ 10^{20}$[72]	$1.4\ 10^{23}$[72]	CUORE	3.9
					$2\ 10^{26}$[20]	CUORE	0.10^{*}
^{136}Xe	0.98	0.030	0.031	$\geq 8.1\ 10^{20}$[20]	$1.2\ 10^{24}$[73]	DAMA	2.3
					$3\ 10^{26}$[20]	XMASS	$0.10^{\dagger}$
					$2\ 10^{27}$[75]	EXO (1t)	$0.055^{\dagger}$
					$4\ 10^{28}$[75]	EXO (10t)	$0.012^{\dagger}$

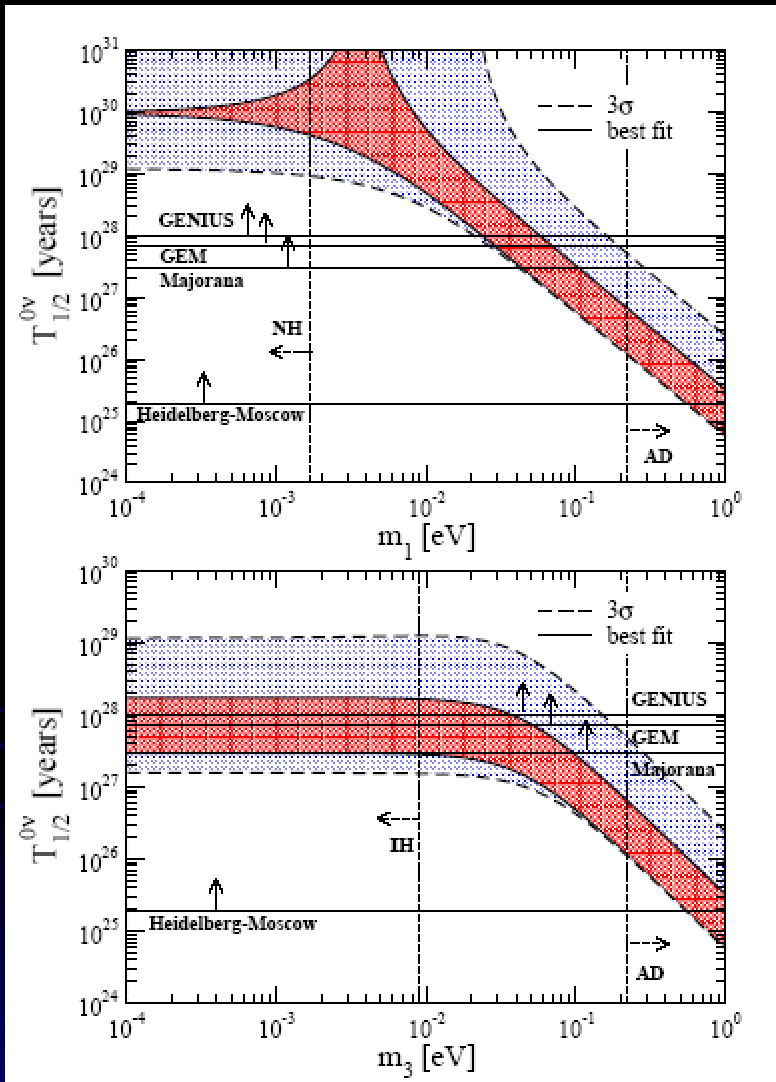
Mee vs lightest m

Normal Hierarchy



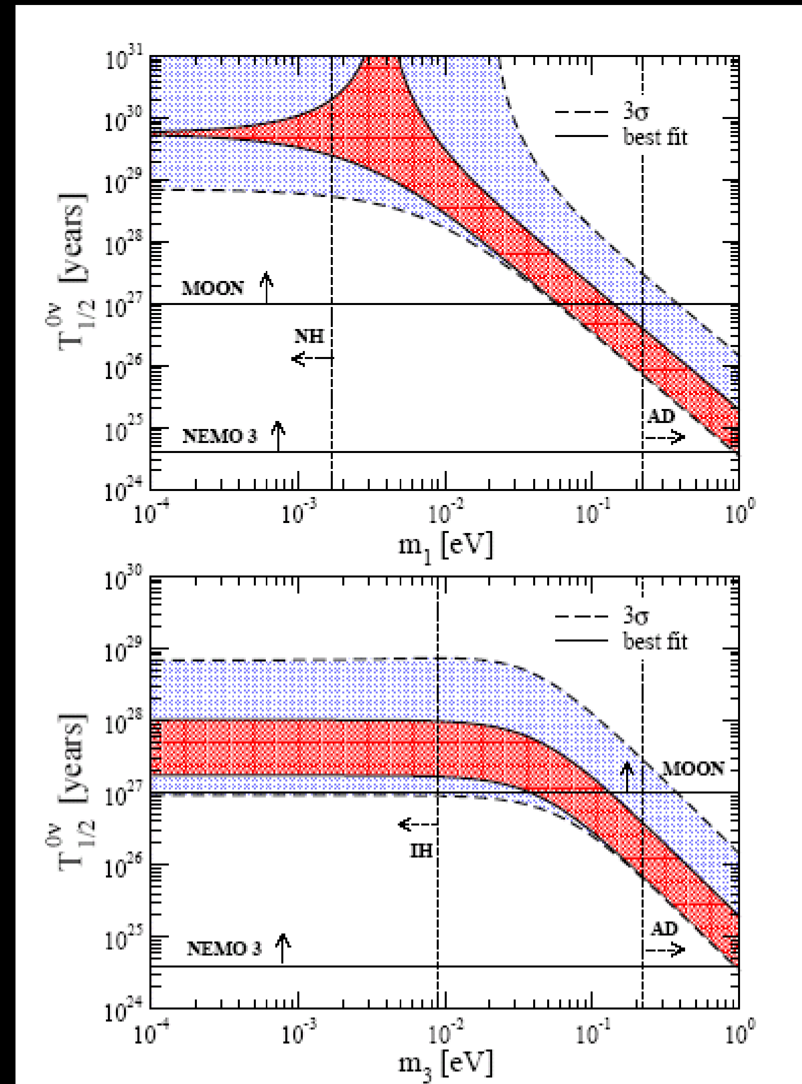
Inverse Hierarchy





3σ
best fit
GENIUS
GEM
Majorana
NH
Heidelberg-Moscow
AD
IH
$T^{0\nu}_{1/2}$ [years]
m_1 [eV]
m_3 [eV]

^{76}Ge



3σ
best fit
MOON
NH
NEMO 3
AD
IH
$T^{0\nu}_{1/2}$ [years]
m_1 [eV]
m_3 [eV]

^{100}Mo

Tritium beta decays

$$3H \rightarrow {}^{3}\mathrm{He} + e^{-} + \bar{\nu}_e \quad (\, m_{\nu e} \text{ limit})$$

$$Q_{\beta} = 18.574 \text{ KeV}$$

Most sensitive to the electron neutrino mass

➔ Since tritium beta-decay has one of the smallest Q-values among all known beta decays:

(1) Superallowed transition between mirror nuclei with a relatively short half-life time (~12.3 years) ➔ An acceptable number of observed events

(2) Atomic structure is less complicated, which leading to a more accurate calculation of atomic effects.

- Kurie Function:

$$K(T) = [\ (Q_\beta - T)\ \sqrt{(Q_\beta - T)^2 - m_{\nu e}^2}\ ]^{1/2}$$

- Mainz and Troitzk experiments:

$$m_{\nu e} < 2.3 \text{ eV (95\% C.L.)}$$

$$m_{\nu e} < 2.5 \text{ eV (95\% C.L.)}$$

With neutrino mixing:

$$3H \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_k \quad (\nu_e = \Sigma_k U_{ek} \nu_k)$$

$$\text{Then } K(T) = [\ (Q_\beta - T)\ \Sigma_k |U_{ek}|^2\ \sqrt{(Q_\beta - T)^2 - m_k^2}\ ]^{1/2}$$

$$m_\beta^2 = \Sigma_k\, |U_{ek}|^2\, m_k^2 = c_{12}^2 c_{13}^2 m_1^2 + s_{12}^2 c_{13}^2 m_2^2 + s_{13}^2 m_3^2$$

$$m_\beta < 2.3 \text{ eV (95\% C.L.)}$$

Summary of Part 1

- Tritium beta decay: Mainz and Troitsk Exp
  $m_1 < 2.2$ eV
- Future Exp. KATRIN:
  sensitivity $m_1 \sim 0.25$ eV

- If the $0\nu\beta\beta$ decay will not observed in future exp. and
  $|m_{\beta\beta}| <$ a few 10^{-2} eV ,
  ➔ Massive neutrinos are either Dirac or Majorana particle, and normal hierarchy

- The observationof the $0\nu\beta\beta$ decay with $|m_{\beta\beta}| > 4.5\ 10^{-2}$ eV will exclude normal hierarchy.

- If the $0\nu\beta\beta$ decay will be observed and

$$0.42\sqrt{\Delta m^2_{atm}} \leq |m_{\beta\beta}| \leq \sqrt{\Delta m^2_{atm}}$$

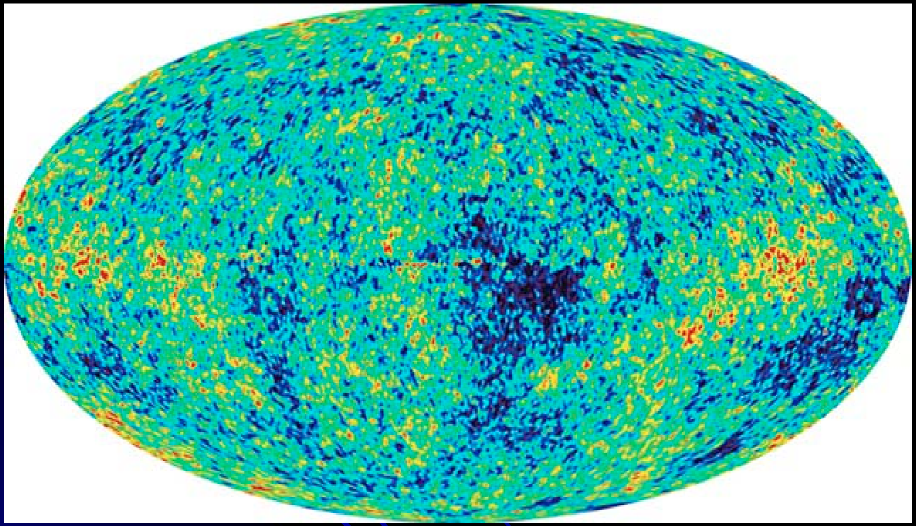
it will be an indication of the inverted hierarchy

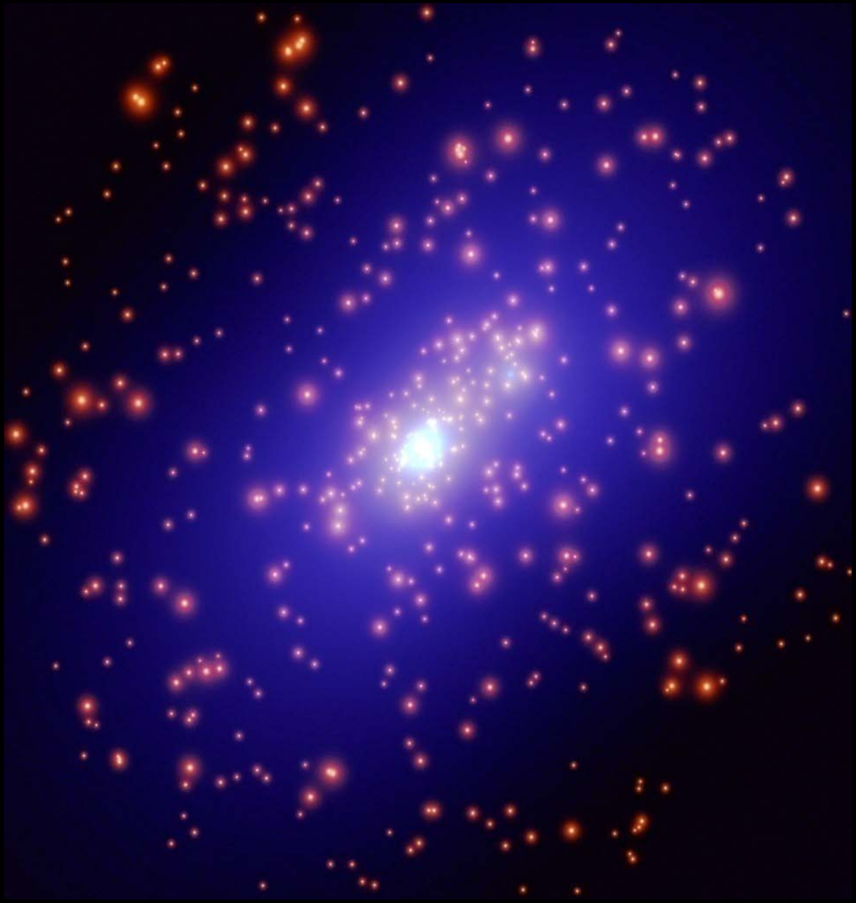
- Normal Hierarchy : M_nu > 0.03 eV
- Inverted Hierarchy: M_nu > 0.07 eV

Remarks: It is really difficult to confirm the normal hierarchy in neutrinoless double beta decay in future experiments.

How can we reach there ? Maybe Long baseline /Magic Baseline Exp. with HE ν-beam (Goswami's talk) or some Astrophysical Observations .

Neutrino Mass bound from Large Scale Structures (CMB, Power Spectrum,.....)





Neutrino Mass in Cosmology

- homogeneous, isotropic universe: place test mass m a distance R from some reference point, assume a mean energy density ρ

$$M(R) = \frac{4}{3}\pi r^3 \rho \qquad H \equiv \frac{1}{R}\frac{dR}{dt}$$

$$E = T + V = \frac{1}{2}m(\frac{dR}{dt})^2 - \frac{GmM(R)}{R} = \frac{1}{2}mR^2(H^2 - \frac{8}{3}\pi\rho G)$$

- so this defines a critical density $\rho_c \equiv \frac{3H^2}{8\pi G} \sim 2 \times 10^{-29}\ \mathrm{g/cm^3}$

- CMB energy density

$$\rho_\gamma = 2\int \frac{d^3q}{(2\pi)^3}\frac{q}{e^{q/T_\gamma} - 1} = \frac{\pi^2}{15}T_\gamma^4 \sim 10^{-5}\rho_c \quad \text{at } T_\gamma \sim 2.72K$$

- ρ_ν for massless νs

$$\rho_\nu = 2\int \frac{d^3q}{(2\pi)^3}\frac{q}{e^{q/T_\nu} + 1} = \frac{7N_\nu}{8} \times \frac{\pi^2}{15}T_\nu^4$$

- relate T_ν to T_γ : initially γs and νs in thermal equilibrium, but decouple when weak interactions drop out of equilibrium. Then γs reheated

$$e^+ + e^- \rightarrow \gamma + \gamma$$

- constant entropy gives after annihilation

$$\frac{T_\gamma}{T_\nu} = \left(\frac{\rho_\gamma + \rho_{e^-} + \rho_{e^+}}{\rho_\gamma} \right)^{1/3} = \left(\frac{11}{4} \right)^{1/3}$$

- ρ_ν for massless νs $\quad \rho_\nu = \rho_\gamma \left(\frac{7N_\nu}{8} \right) \left(\frac{4}{11} \right)^{4/3} \sim 0.7\rho_\gamma$

- for massive neutrinos

$$\frac{n_{\nu_e}}{n_\gamma} = \int \frac{d^3q}{(2\pi)^3} \frac{1}{e^{q/T_\nu} + 1} \Big/ \int \frac{d^3q}{(2\pi)^3} \frac{1}{e^{q/T_\gamma} - 1} = \frac{3}{4} \left(\frac{T_\nu}{T_\gamma} \right)^3 = \frac{3}{11}$$

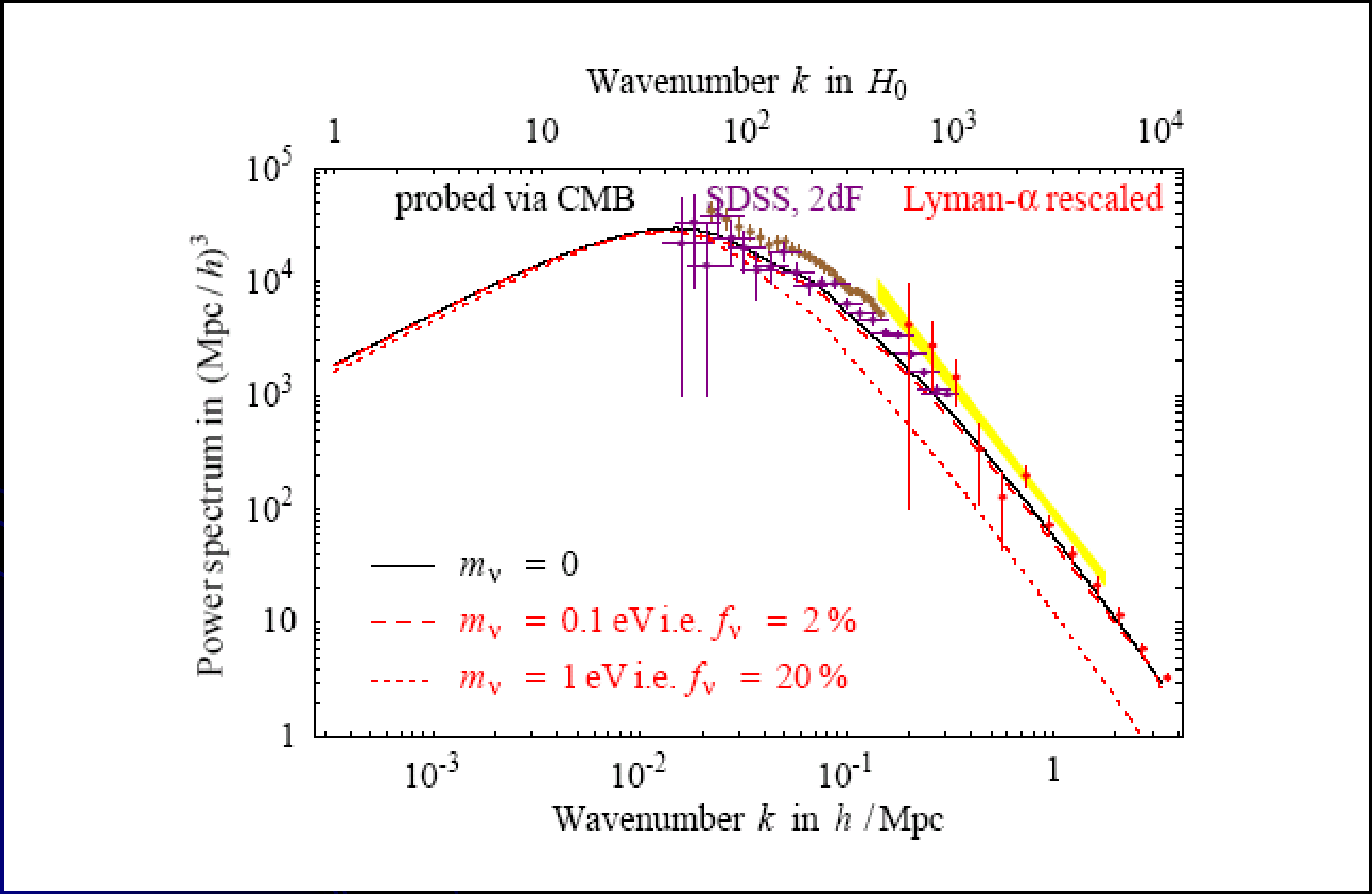
so $\quad \rho_\nu = \frac{3}{11} n_\gamma \sum m_\nu(i) \sim 0.021 \rho_{crit} \sum m_\nu(i)/1 \text{ eV}$

$$\boxed{0.055 \text{ eV} < \sum m_\nu(i) < 6.6 \text{ eV} \;\Rightarrow\; 0.0012 < \rho_\nu/\rho_{crit} < 0.14}$$

Neutrino mass effects

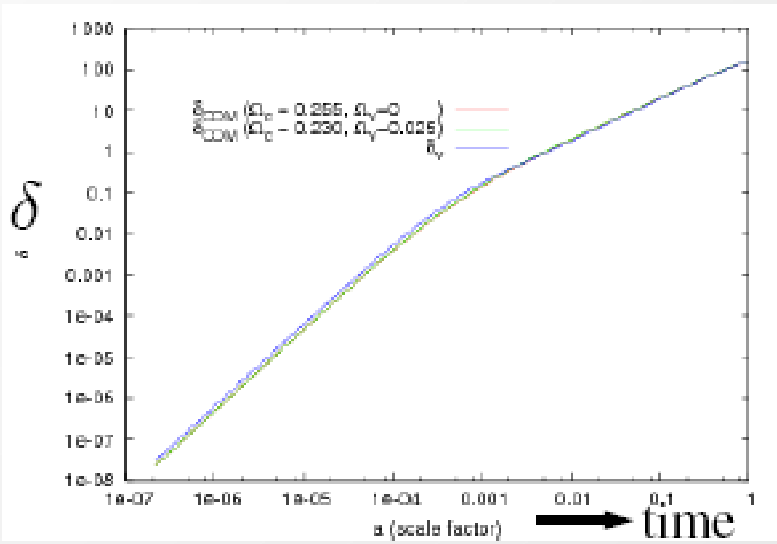
- After neutrinos decoupled from the thermal bath, they stream freely and their density pert. are damped on scale smaller than their free streaming scale.
- The free streaming effect suppresses the power spectrum on scales smaller than the horizon when the neutrino become non-relativistic.
- Δ Pm(k)/Pm(k) = -8 Ω_ν / Ω_m
- Analysis of CMB data are not sensitive to neutrino masses if neutrinos behave as massless particles at the epoch of last scattering. Neutrinos become non-relativistic before last scattering when Ω_ν h^2 > 0.017 (total nu. Masses > 1.6 eV). Therefore the dependence of the position of the first peak and the height of the first peak has a turning point at Ω_ν h^2 = 0.017.

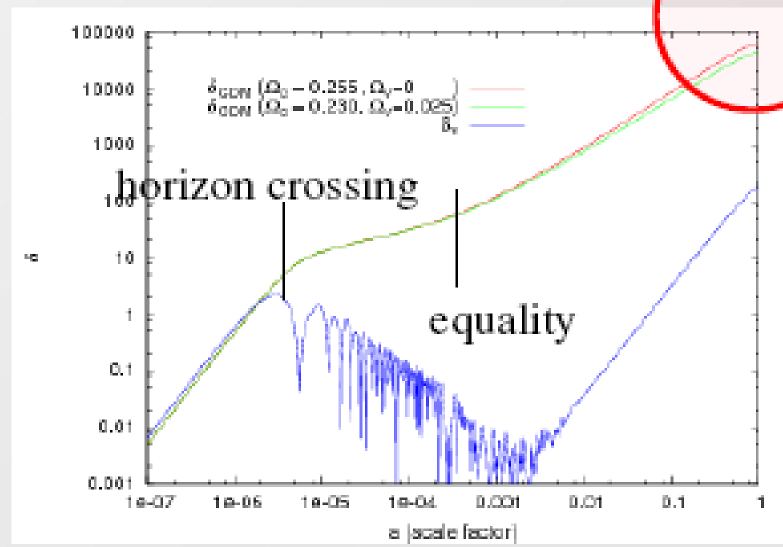
Mass Power spectrum vs Neutrino Masses



Evolution of density perturbations in the presence of massive neutrinos

$k = 3.0 \times 10^{-3}\mathrm{Mpc}^{-1}$ | $k = 3.0\mathrm{Mpc}^{-1}$





$$\delta(x) = \frac{\delta\rho(x)}{\rho} = \int d^3k\, \delta(k) e^{ik\cdot x}$$ Fourier mode of density fluctuation

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\left(\rho_{\mathrm{cdm}}\delta_{\mathrm{cdm}} + \rho_\nu\delta_\nu\right)$$ at large scales

$$\ddot{\delta} + \underbrace{2H\dot{\delta}}_{\text{friction due to expansion}} = 4\pi G\left(\underbrace{\rho_{\mathrm{cdm}}\delta_{\mathrm{cdm}}}_{\text{gravitational force}} + \cancel{\rho_\nu\delta_\nu}\right)$$ at small scales

Power spectrum

$P_m(k,z) = P_*(k) \ \ T^2(k,z)$ Transfer Function:

$T(z,k) := \delta(k,z)/[\delta(k,z=z_*)D(z_*)]$

Primordial matter power spectrum (Ak^n)

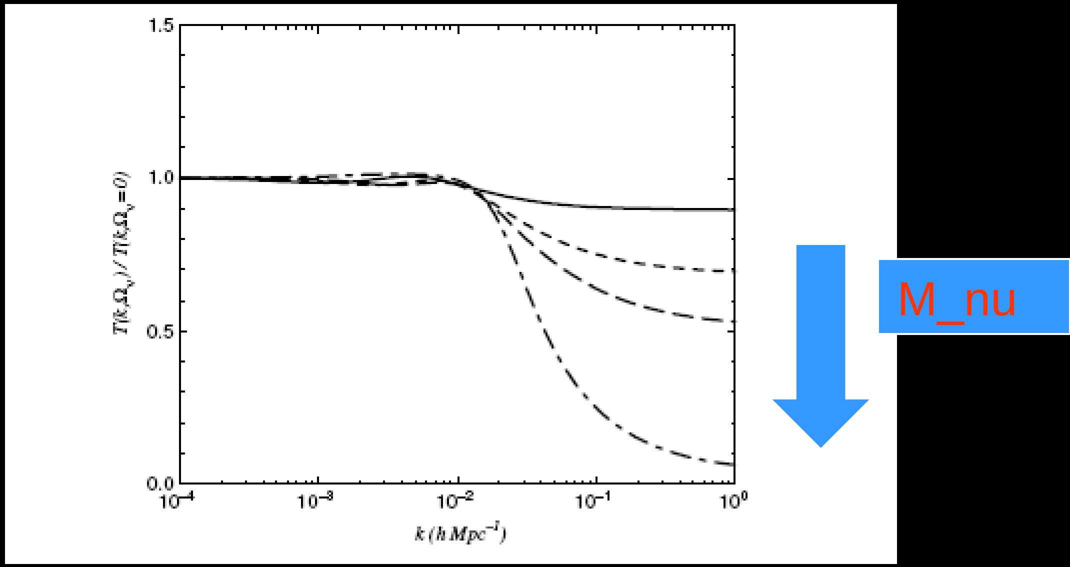
z_*:= a time long before the scale of interested have entered in the horizon

Large scale: $T \sim 1$

Small scale : $T \sim 0.1$

$\Delta P_m(k)/P_m(k) \ \sim -8\ \Omega_\nu/\Omega_m$

$= -8\ f_\nu$



Numerical Analysis

MCMC likelihood analysis

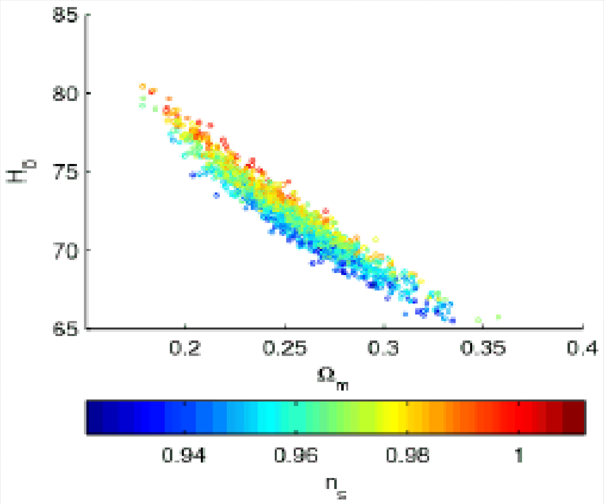
- cosmological parameters (7 params)

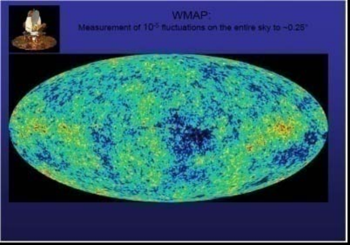
$$\vec{P} = (\Omega_b h^2, \Omega_c h^2, \theta, \tau, m_\nu, n_s, A_s)$$

- explore the likelihoods of WMAP5 and CFHTLS data using Markov Chain Monte Calro sampling

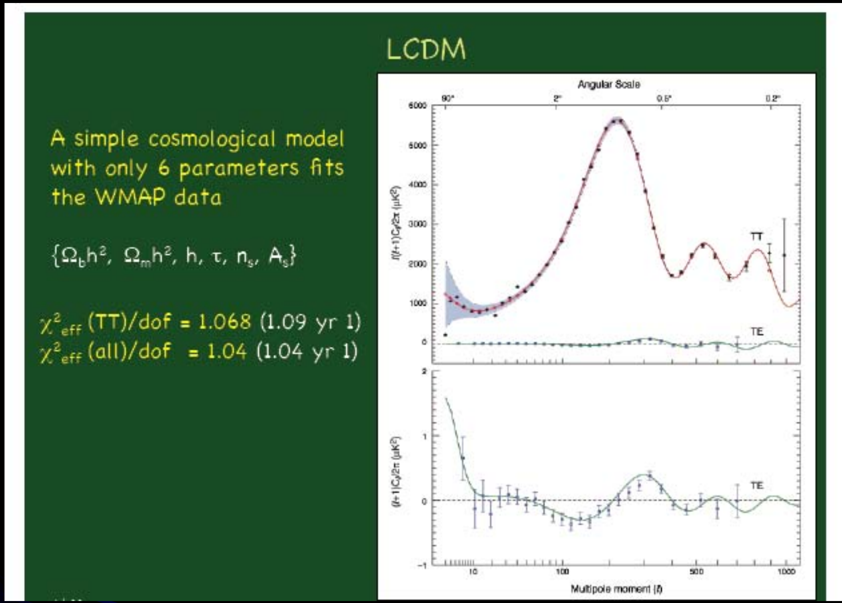
- CosmoMC:
  Cosmological MCMC engine

(http://cosmologist.info/cosmomc)

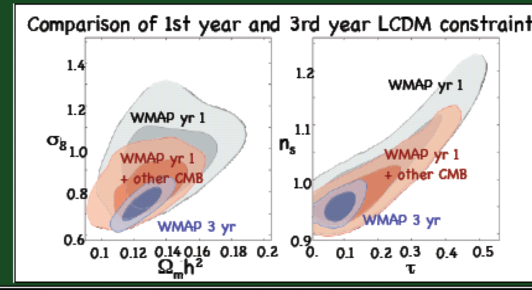




Experimental Obs.(WMAP+PLANCK)



Improvement in Parameters from EE tau measurement



Parameter	First Year Mean	WMAPext Mean	Three Year Mean	First Year ML	WMAPext ML	Three Year ML
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	2.23 ± 0.08	2.30	2.21	2.22
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	0.126 ± 0.009	0.145	0.138	0.128
H_0	72^{+5}_{-5}	73^{+3}_{-3}	74^{+3}_{-3}	68	71	73
τ	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	0.093 ± 0.029	0.10	0.10	0.092
n_s	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	0.961 ± 0.017	0.97	0.96	0.958
Ω_m	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	0.234 ± 0.035	0.32	0.27	0.24
σ_8	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	0.76 ± 0.05	0.88	0.82	0.77

n_s: Spectral index
tau: optical depth
sigma_8: rms fluctuation parameter
A_s: the amp. of the primordial scalar power spectrum

$$\sigma_8 = \langle \left(\frac{\delta M}{M}\right)^2 \rangle = \int dk\, 4\pi k^2 P(k) [3 \frac{\sin(kr) - kr\cos(kr)}{(kr)^2}]^2$$

Within Standard Cosmology Model (LCDM)

Upper limits on neutrino masses from Cosmology

Assume that the underlying cosmological model is:

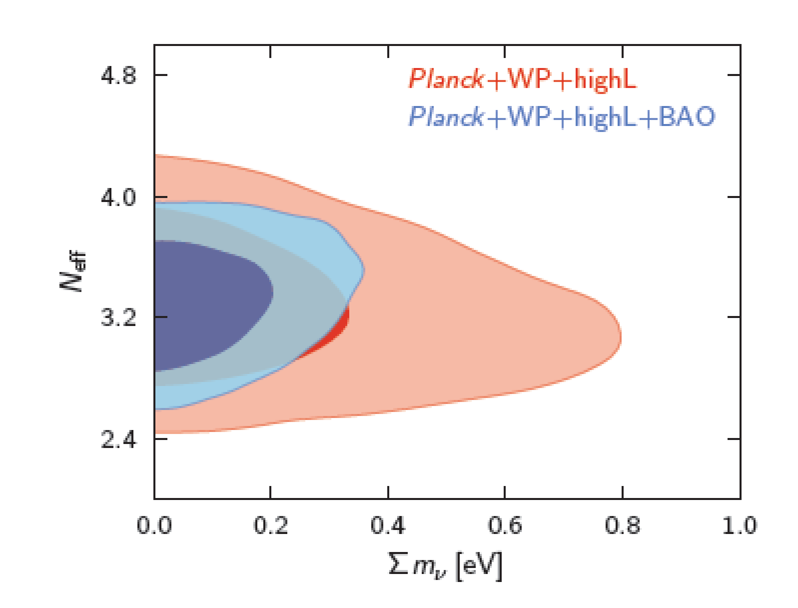
- the standard spatially flat ΛCDM model with adiabatic primodial perturbations,
- they have no non-standard interactions,
- they decouple from the thermal background at temperatures of order 1 MeV,
- use the relation between the sum of the neutrino masses and their contribution to the energy density of the universe is:

$$\Omega_\nu \ h^2 = M_\nu / 93.14 \ eV \qquad (1)$$

Data	Authors	M_ν-bound
2dFGRS (P01)	Elgarøy et al. [2002]	1.8 eV
CMB+2dFGRS(C05)	Sanchez et al. [2005]	1.2 eV
CMB+LSS+SNIa+BAO	Goobar et al. [2006]	0.62 eV
WMAP (3 year) alone	Fukugita et al. [2006]	2.0 eV
CMB+LSS+SNIa	Spergel et al. [2006]	0.68 eV
CMB + LSS+SNIa+BAO+Lyα	Seljak et al. [2006]	0.17 eV

Table 2: Some recent cosmological neutrino mass bounds (95 % CL).

Planck 2013 Results



	Planck+WP		*Planck*+WP+BAO		*Planck*+WP+highL		*Planck*+WP+highL+BAO	
Parameter	Best fit	95% limits	Best fit	95% limits	Best fit	95% limits	Best fit	95% limits
Ω_K	-0.0326	$-0.037^{+0.043}_{-0.049}$	0.0006	$0.0000^{+0.0066}_{-0.0067}$	-0.0389	$-0.042^{+0.043}_{-0.048}$	-0.0003	$-0.0005^{+0.0065}_{-0.0066}$
Σm_ν [eV]	0.002	< 0.933	0.000	< 0.247	0.000	< 0.663	0.001	< 0.230
N_{eff}	3.25	$3.51^{+0.80}_{-0.74}$	3.32	$3.40^{+0.59}_{-0.57}$	3.38	$3.36^{+0.68}_{-0.64}$	3.33	$3.30^{+0.54}_{-0.51}$
Y_P	0.2896	$0.283^{+0.045}_{-0.048}$	0.2889	$0.283^{+0.043}_{-0.045}$	0.2652	$0.266^{+0.040}_{-0.042}$	0.2701	$0.267^{+0.038}_{-0.040}$
$dn_s/d\ln k$	-0.0125	$-0.013^{+0.018}_{-0.018}$	-0.0097	$-0.013^{+0.018}_{-0.018}$	-0.0146	$-0.015^{+0.017}_{-0.017}$	-0.0143	$-0.014^{+0.016}_{-0.017}$
$r_{0.002}$	0.000	< 0.120	0.000	< 0.122	0.000	< 0.108	0.000	< 0.111
w	-1.94	$-1.49^{+0.65}_{-0.57}$	-1.106	$-1.13^{+0.24}_{-0.25}$	-1.94	$-1.51^{+0.62}_{-0.53}$	-1.113	$-1.13^{+0.23}_{-0.25}$

Questions:

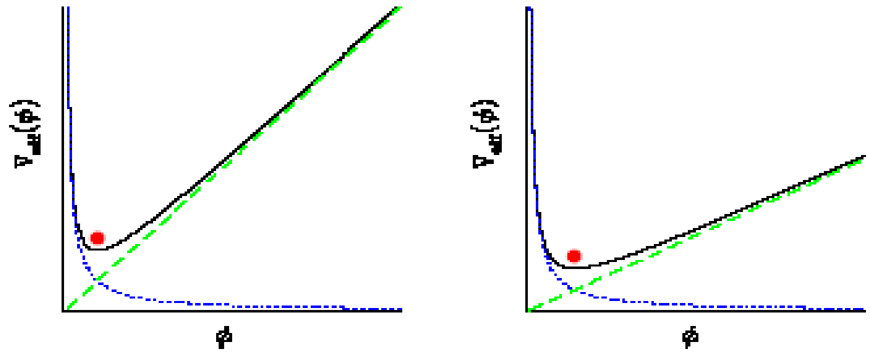
issues a bit farther from resolution;

it is known that the dark energy density ~ m_ν^4

→ is this an accident , or not ?

What is the upper bound of neutrino masses beyond ΛCDM Model ?

Example: Interacting Neutrino-Dark-Energy Model



At low energy,

$$\mathcal{L} = m_\nu \bar{\nu}^c \nu + V_{tot}(m_\nu) \, ,$$

$n_\nu \, m_\nu(\phi)$

Scalar potential in vacuum

$$\text{where } V_{tot}(m_\nu) = V_\nu(m_\nu) + V_0(m_\nu)$$

The condition of minimization of V_{tot} determines the physical neutrino mass.

Background Equations:

K. Ichiki and YYK:2007

Equations for quintessence scalar field are given by

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 \frac{dV_{\rm eff}(\phi)}{d\phi} = 0\,, \qquad (1)$$

$$V_{\rm eff}(\phi) = V(\phi) + V_{\rm I}(\phi)\,, \qquad (2)$$

$$V_{\rm I}(\phi) = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2(\phi)} f(q)\,, \qquad (3)$$

$$m_\nu(\phi) = \bar{m}_i e^{\beta \frac{\phi}{M_{\rm pl}}} \text{ (as an example)}, \qquad (4)$$

Energy densities of mass varying neutrino (MVN) and quintessence scalar field are described as

$$\rho_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \sqrt{q^2 + a^2 m_\nu^2} f_0(q)\,, \qquad (5)$$

$$3P_\nu = a^{-4} \int \frac{d^3q}{(2\pi)^3} \frac{q^2}{\sqrt{q^2 + a^2 m_\nu^2}} f_0(q)\,, \qquad (6)$$

$$\rho_\phi = \frac{1}{2a^2}\dot{\phi}^2 + V(\phi)\,, \qquad (7)$$

$$P_\phi = \frac{1}{2a^2}\dot{\phi}^2 - V(\phi)\,. \qquad (8)$$

From equations (5) and (6), the equation of motion for the background energy density of neutrinos is given by

$$\dot{\rho}_\nu + 3\mathcal{H}(\rho_\nu + P_\nu) = \frac{\partial \ln m_\nu}{\partial \phi} \dot{\phi} (\rho_\nu - 3P_\nu)\,. \qquad (9)$$

Perturbation Equations:

We consider the linear perturbation in the synchronous Gauge and the linear elements:

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]\,, \qquad (10)$$

1 Boltzmann Equation for Mass Varying Neutrino

Here we have splitted the comoving momentum q_j into its magnitude and direction: $q_j = qn_j$, where $n^i n_i = 1$.

The Boltzmann equation is

$$\frac{Df}{D\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = \left(\frac{\partial f}{\partial \tau}\right)_C \quad . \tag{34}$$

in terms of these variables. From the time component of geodesic equation,

$$\frac{1}{2}\frac{d}{d\tau}\left(P^0\right)^2 = -\Gamma^0_{\alpha\beta}P^\alpha P^\beta - mg^{0\nu}m_{,\nu} \ , \tag{35}$$

and the relation $P^0 = a^{-2}\epsilon = a^{-2}\sqrt{q^2 + a^2 m_\nu^2}$, we have

$$\frac{dq}{d\tau} = -\frac{1}{2}\dot{h}_{ij}qn^i n^j - a^2\frac{m}{q}\frac{\partial m}{\partial x^i}\frac{dx^i}{d\tau} \ . \tag{36}$$

We will write down each term up to $\mathcal{O}(h)$:

$$\begin{aligned}
\frac{\partial f}{\partial \tau} &= \frac{\partial f_0}{\partial \tau} + f_0\frac{\partial \Psi}{\partial \tau} + \frac{\partial f_0}{\partial \tau}\Psi \\
\frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} &= \frac{q}{\epsilon}n^i \times f_0\frac{\partial \Psi}{\partial x^i} \ ,
\end{aligned}$$

Then we obtain the hierarchy for MVN

$$
\begin{aligned}
\dot{\Psi}_0 &= -\frac{q}{\epsilon}k\Psi_1 + \frac{\dot{h}}{6}\frac{\partial \ln f_0}{\partial \ln q} \ , && (43)\\
\dot{\Psi}_1 &= \frac{1}{3}\frac{q}{\epsilon}k\left(\Psi_0 - 2\Psi_2\right) + \kappa \ , && (44)\\
\dot{\Psi}_2 &= \frac{1}{5}\frac{q}{\epsilon}k(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{\partial \ln f_0}{\partial \ln q} \ , && (45)\\
\dot{\Psi}_\ell &= \frac{q}{\epsilon}k\left(\frac{\ell}{2\ell+1}\Psi_{\ell-1} - \frac{\ell+1}{2\ell+1}\Psi_{\ell+1}\right) \ . && (46)
\end{aligned}
$$

where

$$\kappa = -\frac{1}{3}\frac{q}{\epsilon}k\frac{a^2m^2}{q^2}\delta\phi\frac{\partial \ln m_\nu}{\partial \phi}\frac{\partial \ln f_0}{\partial \ln q} \ . \tag{47}$$

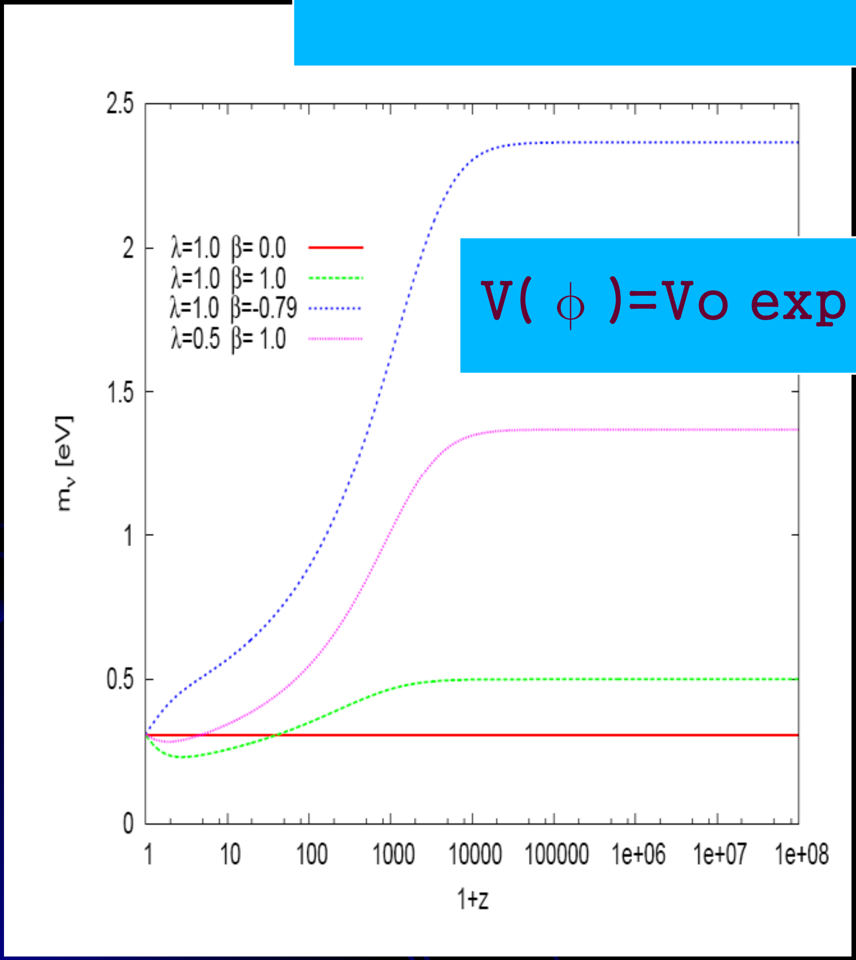
Here we used the recursion relation

$$(\ell+1)P_{\ell+1}(\mu) = (2\ell+1)\mu P_\ell(\mu) - \ell P_{\ell-1}(\mu) \ . \tag{48}$$

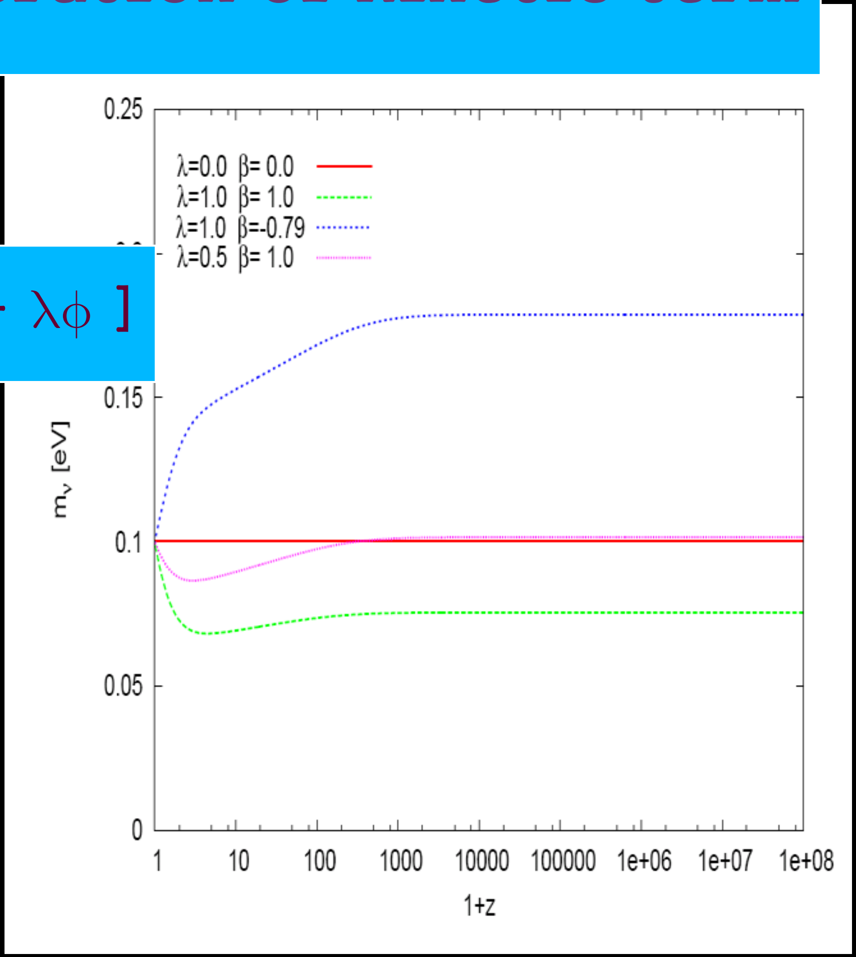
We have to solve these equations with a q-grid for every wavenumber k...

Varying Neutrino Mass

With full consideration of Kinetic term



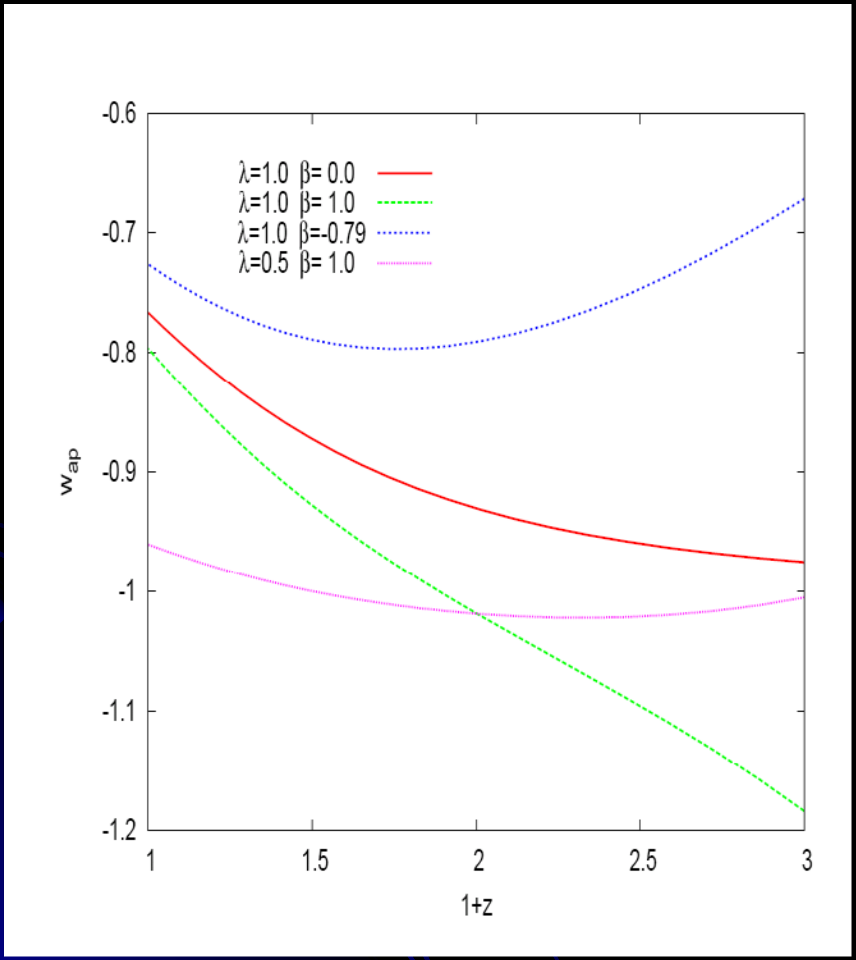
$V(\phi)=V_o \exp[-\lambda\phi]$



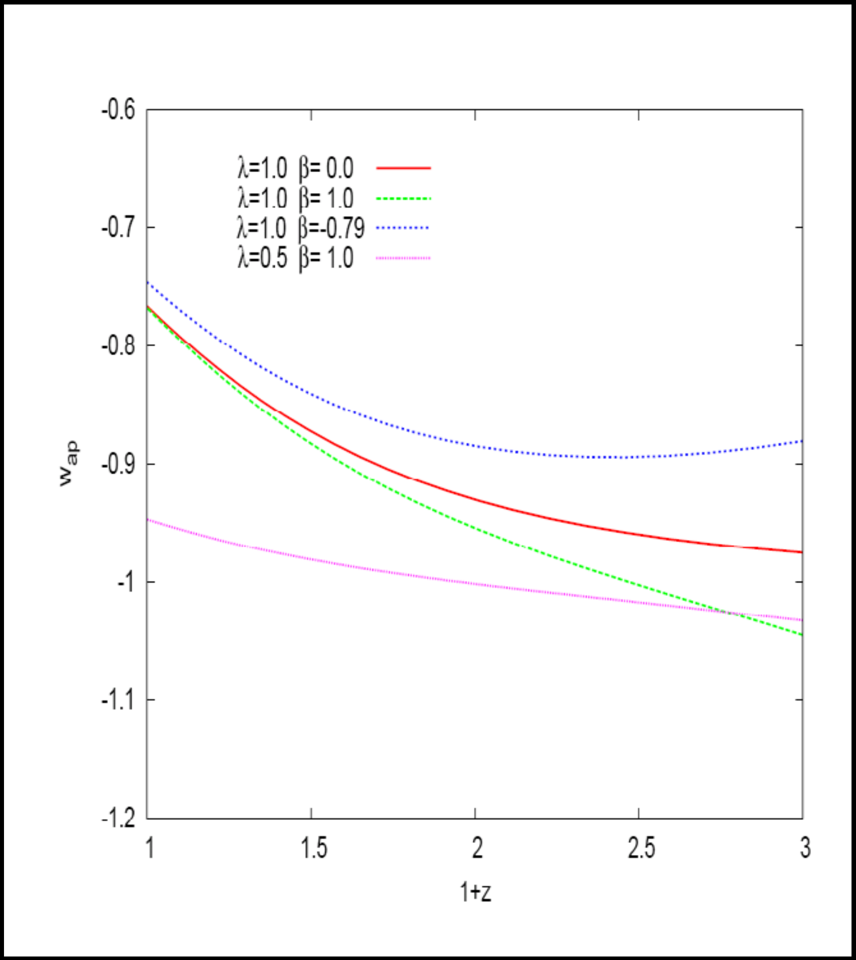
$M\nu=0.9$ eV

$M\nu=0.3$ eV

W_eff



$M\nu = 0.9$ eV



$M\nu = 0.3$ eV

Neutrino Masses vs z

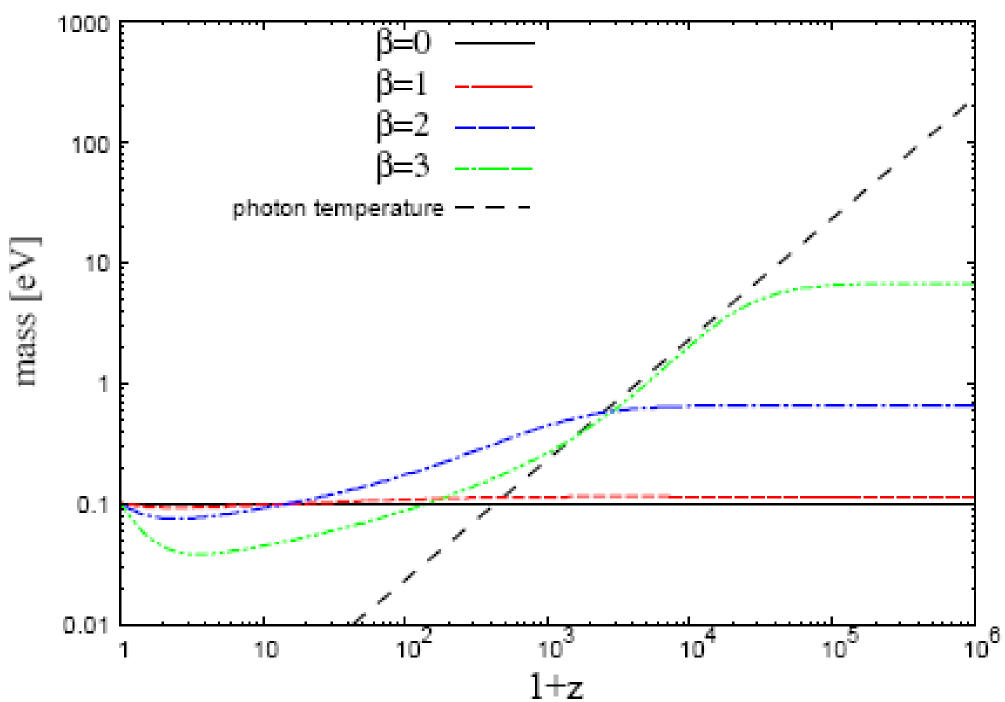
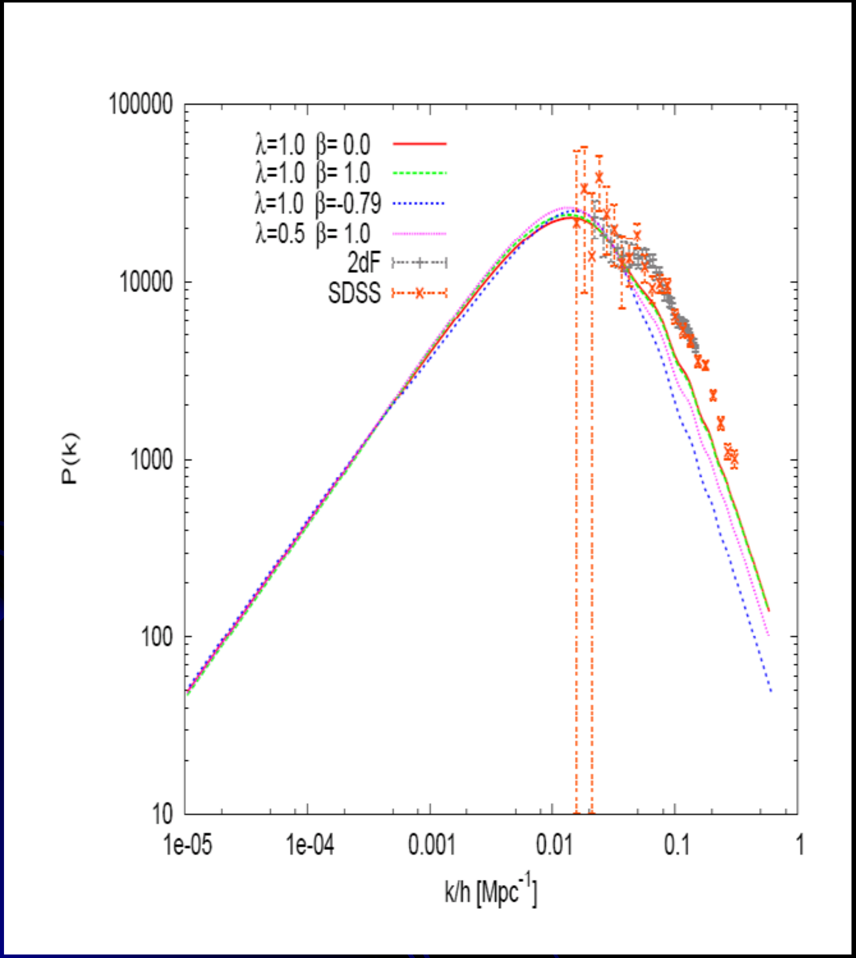
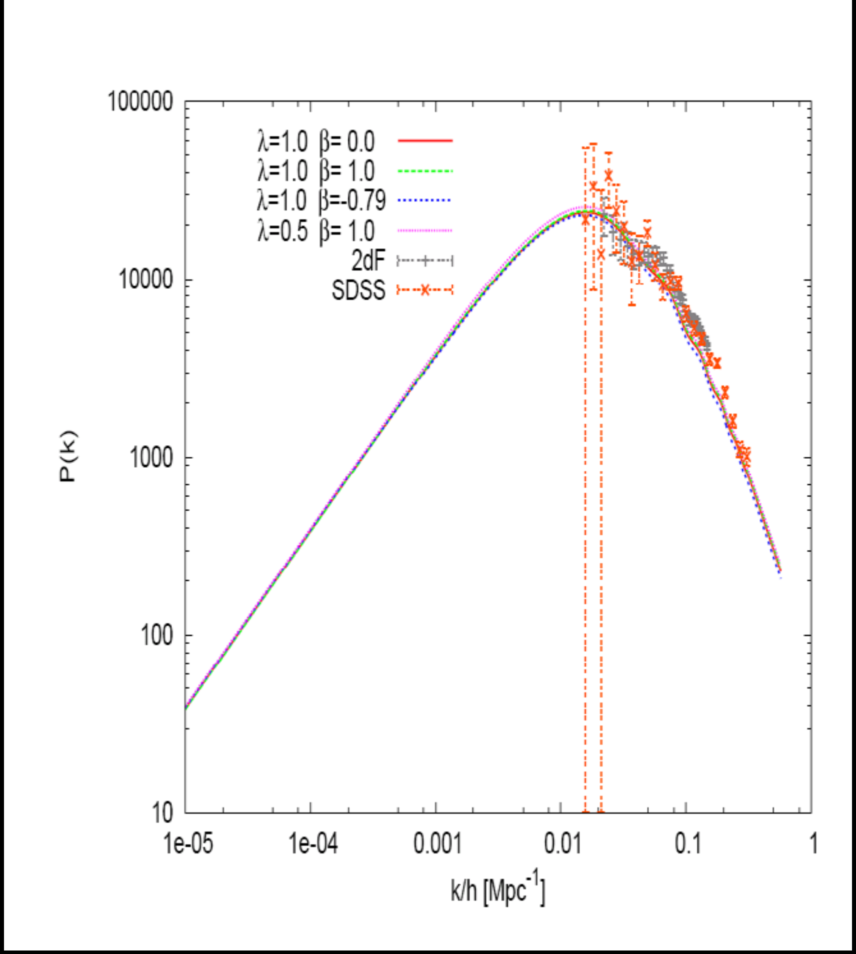


Figure 5: Examples of the time evolution of neutrino mass in power law potential models (Model I) with $\alpha = 1$ and $\beta = 0$ (black solid line), $\beta = 1$ (red dashed line), $\beta = 2$ (blue dash-dotted line), $\beta = 3$ (dash-dot-dotted line). The larger coupling parameter leads to the larger mass in the early universe.

Power-spectrum (LSS)

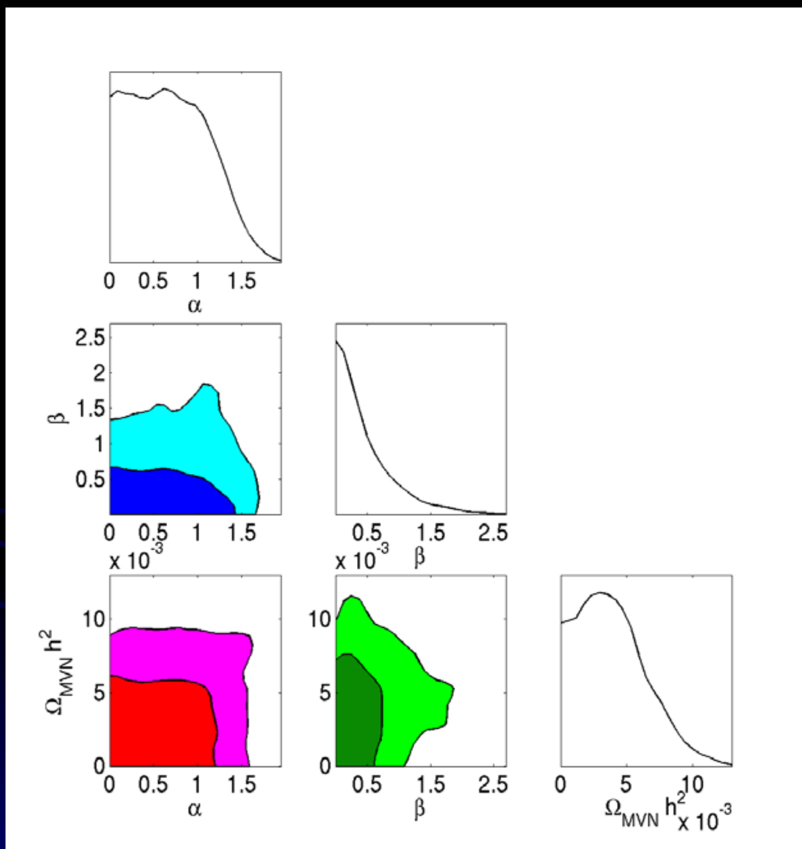


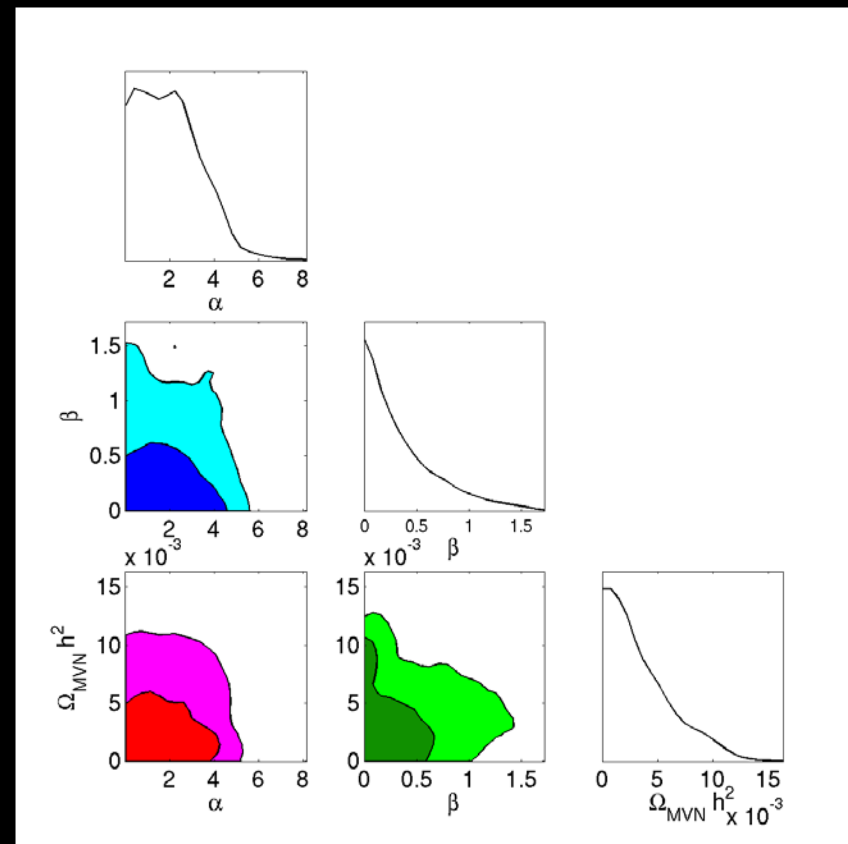


$M\nu = 0.9$ eV

$M\nu = 0.3$ eV

Constraints from Observations





$$V(\phi) = V_0 e^{-\alpha\phi}$$

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

	Exponential	Potential	Inverse-Power	Potential
Quantities	Means	1σ	Means	1σ
$\Omega_B h^2(10^2)$	2.21 ± 0.07	$2.15 - 2.28$	2.21 ± 0.07	$2.15 - 2.28$
$\Omega_{CDM} h^2(10^2)$	11.10 ± 0.63	$10.48 - 11.72$	11.10 ± 0.62	$10.52 - 11.68$
H_0	65.61 ± 3.26	$62.37 - 68.78$	65.97 ± 3.61	$62.30 - 69.37$
Z_{re}	11.07 ± 2.44	$10.07 - 12.35$	10.87 ± 2.58	$9.81 - 12.15$
α	0.70 ± 0.42	< 0.92	2.08 ± 1.35	< 2.63
β	0.50 ± 0.48	< 0.58	0.38 ± 0.35	< 0.46
$M_{\nu 0}(eV)$	0.047 ± 0.046	< 0.055	0.057 ± 0.070	< 0.051
n_s	0.95 ± 0.02	$0.94 - 0.97$	0.95 ± 0.02	$0.94 - 0.97$
$A_s(10^{10})$	20.72 ± 1.24	$19.47 - 21.95$	20.66 ± 1.31	$19.38 - 21.92$
$\Omega_Q(10^2)$	68.22 ± 4.17	$64.38 - 72.08$	68.54 ± 4.81	$64.02 - 72.94$
$Age/Gyrs$	13.69 ± 0.19	$13.77 - 14.15$	13.95 ± 0.20	$13.76 - 14.15$
$\Omega_{MVN} h^2(10^2)$	0.38 ± 0.25	< 0.48	0.36 ± 0.29	< 0.44
τ	0.09 ± 0.03	$0.06 - 0.11$	0.08 ± 0.03	$0.05 - 0.11$

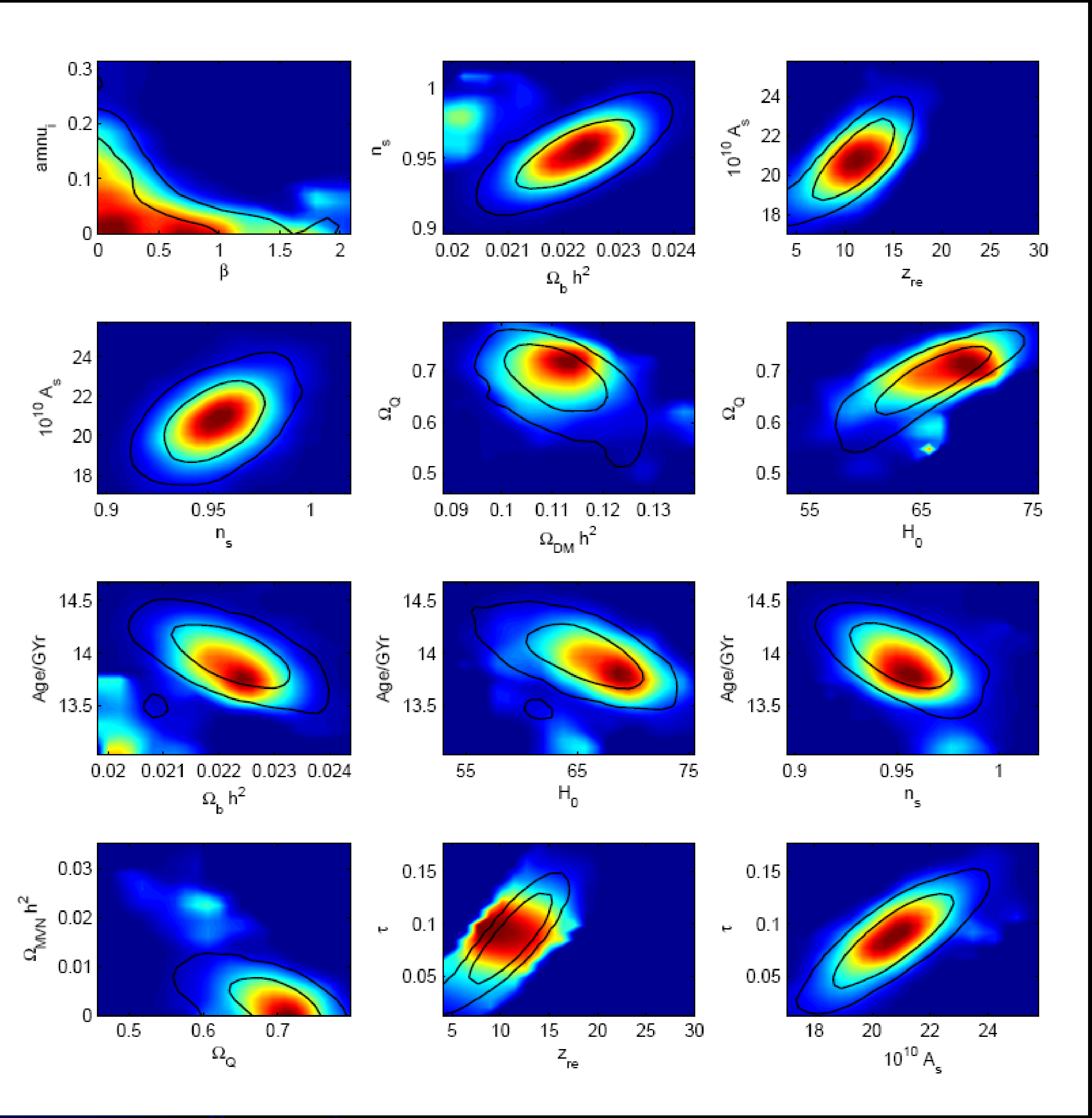
Table 3: Global Fit analysis data using usual choice of potentials and coupling: $V(\phi) = V_0 e^{-\alpha\phi}$, $M^{4+\alpha}/\phi^\alpha$ and $m_\nu(\phi) = M_{\nu 0} e^{\beta\phi}$

Neutrino mass Bound: M_ν < 0.87 eV @ 95 % C.L.

Cosmological parameters after Planck obs. @2013

	Planck+WP	*Planck*+WP+BAO	*WMAP*-9
$\Omega_b h^2$	0.02206 ± 0.00028	0.02220 ± 0.00025	0.02309 ± 0.00130
$\Omega_c h^2$	0.1174 ± 0.0030	0.1161 ± 0.0028	0.1148 ± 0.0048
τ	0.095 ± 0.014	0.097 ± 0.014	0.089 ± 0.014
H_0	65.2 ± 1.8	66.7 ± 1.1	74 ± 11
n_s	0.974 ± 0.012	0.975 ± 0.012	0.973 ± 0.014
$\log(10^{10} A_s)$	3.106 ± 0.029	3.100 ± 0.029	3.090 ± 0.039
α/α_0	0.9936 ± 0.0043	0.9989 ± 0.0037	1.008 ± 0.020

Good Agreement !



amnu_i
β
n_s
Ω_b h^2
10^10 A_s
z_re
Ω_Q
Ω_DM h^2
H_0
Age/GYr
Ω_MVN h^2
τ

Summary: Neutrino Mass Bounds in Interacting Neutrino DE Model

Without Ly-alpha Forest data (only 2dFGRS + HST + WMAP5)

- Omega_nu h^2 < 0.0044 ; 0.0095 (inverse power-law potential)
- < 0.0048 ; 0.0090 (sugra type potential)
- < 0.0048 ; 0.0084 (exponential type potential)

provides the total neutrino mass bounds

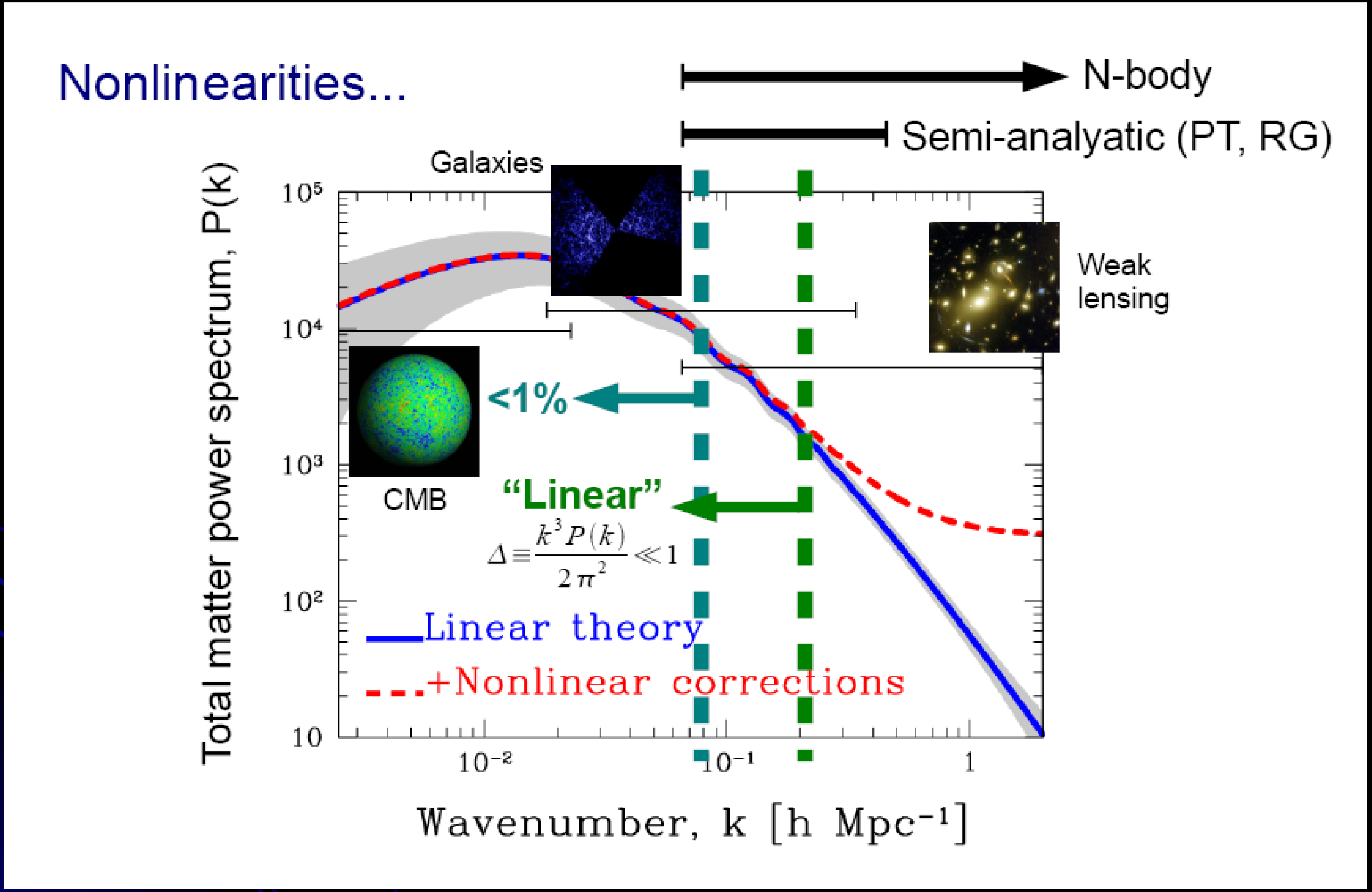
M_nu < 0.45 eV (68 % C.L.)
< 0.87 eV (95 % C.L.)

Including Ly-alpah Forest data

Omega_nu h^2 < 0.0018; 0.0046 (sugra type potential)
corresponds to
M_nu < 0.17 eV (68 % C.L.)
< 0.43 eV (95 % C.L.)

We have weaker bounds in the interacting DE models

Nonlinear Effects



Future Prospects:

- error on LSS surveys of power amplitude $\propto 1/\sqrt{N}$, N number of modes
- higher z surveys sample larger volumes;
  high-z data from linear epoch in structure growth, simplifying analysis
- additional constraints on large scales from CMB
- and on small scales from Lyman alpha forest samples for z<6

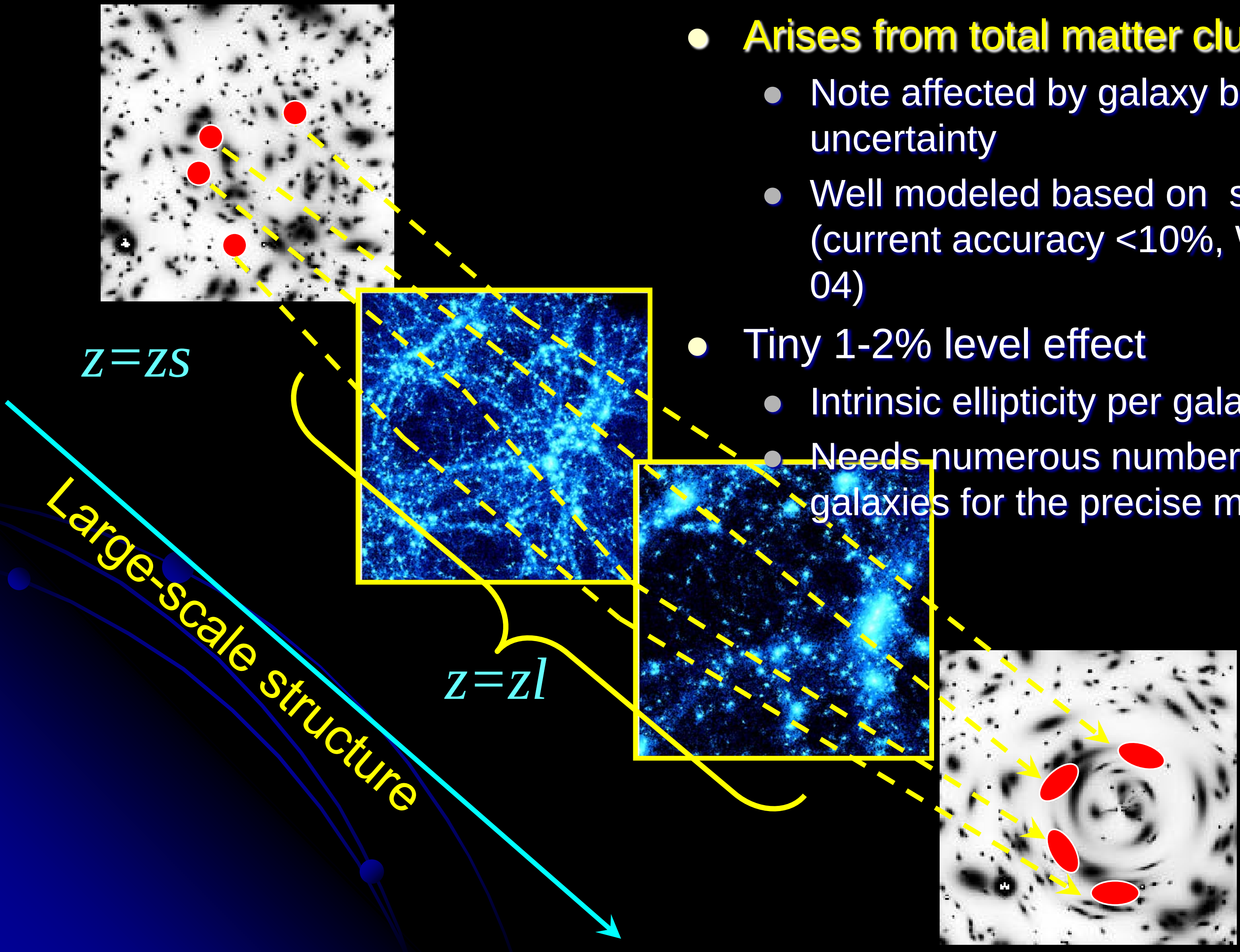
  - SDSS: 3000 QSOs at 2.2<z<4.4 - 15% errors, 12 wave numbers

  - SDSS-III BOSS survey goal is 160,000 QSOs by 2015

  - 10^5 QSO survey + Planck CMB data: 0.05 eV at 1σ

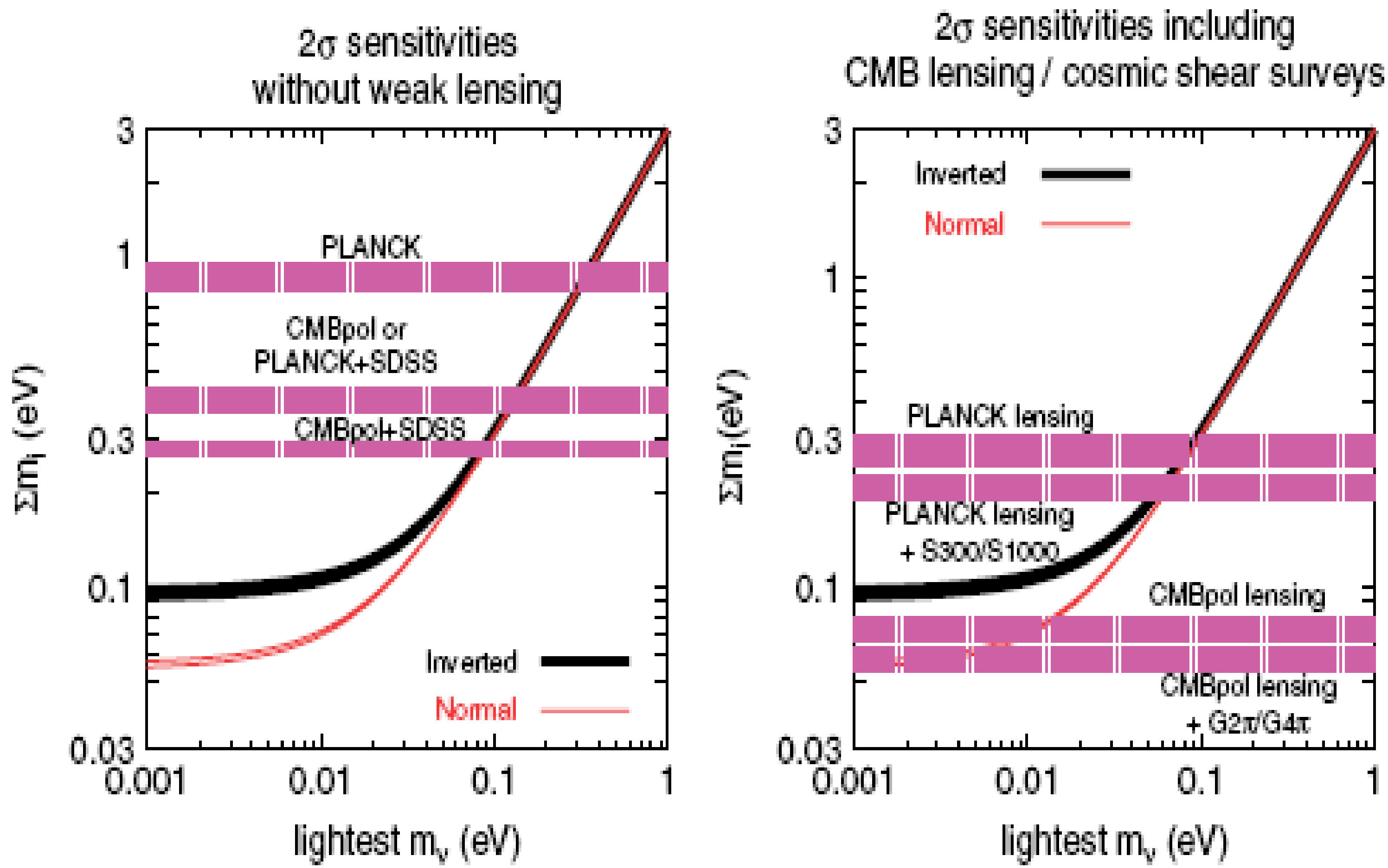
  - 21cm: radio telescope 0.1 km^2 power to 1% $0.03 < k < 0.7$/Mpc
    1.0 km^2 sensitive to 0.05 eV at $> 7\sigma$
- weak lensing, medium-small scales: ground-based survey over 70% of sky sampling 30 galaxies/arc-minute + Planck predicted to reach 0.04 eV

Cosmological weak lensing



- Arises from total matter clustering
  - Note affected by galaxy bias uncertainty
  - Well modeled based on simulations (current accuracy <10%, White & Vale 04)
- Tiny 1-2% level effect
  - Intrinsic ellipticity per galaxy, ~30%
  - Needs numerous number (10^8) of galaxies for the precise measurement

Future Prospects from Astrophysical Observations



Summary

- LCDM model provides

M_nu < 0.6 - 0.7 eV (LSS + CMB +BAO)

< 0.23 - 0.93 eV (Planck + WP + High L +BAO)

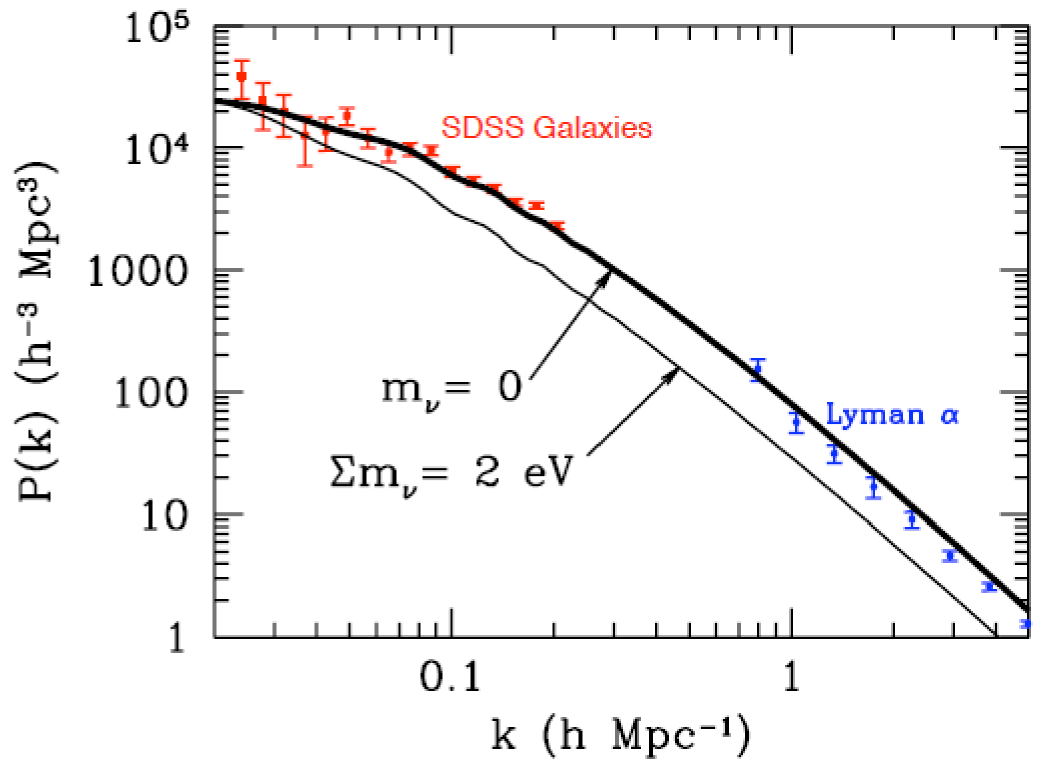
< 0.2-0.3 eV (including Lya data)

- Interacting Neutrino Dark-Energy Model provides more weaker bounds:

M_nu < 0.8 - 0.9 eV (LSS + CMB)

< 0.4 - 0.5 eV (including Lya data)

- Lya-forest data includes the uncertainty from
  - continuum errors
  - unidentified metal lines
  - noise



APS
MultiDivisional
Neutrino Study

But good to remain cautious: long way to go
systematics issues in combining data sets
parameter degeneracies, e.g., w

Summary of Methods to Obtain Neutrino Masses

Single beta decay	$\Sigma_\iota\ m_i^2\ \lvert U_{ei}\rvert^2$	Sensitivity 0.2 eV
Double beta decay	$m_{\beta\beta} = \lvert \Sigma_\iota\ m_i\ \lvert U_{ei}\rvert^2\ \varepsilon_i \rvert$ ε_i = Majorana phases	Sensitivity 0.01 eV
Neutrino oscillations	$\delta m^2 = m_1^2 - m_2^2$	Observed $\sim 10^{-5}$ eV^2
Cosmology	$\Omega_\nu \rightarrow \Sigma_\iota\ m_i$	Observed ~ 0.1 eV

Only double beta decay is sensitive to Majorana nature.

THANK YOU!!

Birthday Greetings

Thanks

For

your attention!

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