Predictions From High Scale Mixing Unification Hypothesis

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Outline

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Introduction

- Neutrinos are probably the most mysterious and ill understood of all known particles
- In past neutrinos have thrown up quite a few surprises: They still keep on surprising us !!
- Recent measurements conclusively show $\theta_{13} \neq 0$[1]: The latest "surprise"
- Measurement of θ_{13} was long awaited: Provides crucial test of several candidate models
- Is there a "natural" way of understanding non-zero and "relatively large" θ_{13}?
- In this talk we discuss one such possibility: **High Scale Mixing Unification (HSMU)**

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Current Experimental Scenario

- Global Fits for neutrino oscillation parameters[2]:

Quantity	Best Fit $\pm$1-σ	3-σ Range
$\theta_{12}/^\circ$	$33.36^{+0.81}_{-0.78}$	31.09 – 35.89
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.3}_{-1.3}$	35.8 – 54.8
$\theta_{13}/^\circ$	$8.66^{+0.44}_{-0.46}$	7.19 – 9.96
$\delta_{\rm CP}/^\circ$	300^{+66}_{-138}	0 – 360
Δm^2_{21} $(10^{-5}$ eV$^2)$	$7.50^{+0.18}_{-0.19}$	7.00 – 8.09
Δm^2_{31} $(10^{-3}$ eV$^2)$ (N)	$2.473^{+0.070}_{-0.067}$	2.276 – 2.695
Δm^2_{23} $(10^{-3}$ eV$^2)$ (I)	$2.427^{+0.042}_{-0.065}$	2.242 – 2.649

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Open Questions

- Despite tremendous amount of theoretical and experimental research our understanding of neutrinos is still poor
- Nature of neutrinos: Dirac or Majorana?
- Mass of neutrinos: Hierarchical or quasi degenerate?
- Mass Hierarchy: Normal or Inverted?
- CP violation: $\delta_{\rm CP}$?
- Octant of θ_{23}: $\theta_{23} < 45^\circ$ or $\theta_{23} > 45^\circ$?
- Why lepton and quark mixing parameters are so different?

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- Has lead to much advancement in our understanding: Electro-Magnetism, Electro-Weak force etc
- Current research: Unification of forces
- Grand Unified Theories (GUTs): Unification of gauge couplings
- Key Ingredient: Quarks and Leptons in same multiplet
- Flavor structure of quarks and leptons: Not totally disconnected
- Interesting possibility: "High Scale" Unification of CKM and PMNS mixing parameters

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Unification of CKM and PMNS mixing parameters

- How is this possible?
- Numerically they are so different from each other!!
- The quark mixing matrix[3]

$$|U_{\rm CKM}|_{3\sigma} =$$

$$\begin{pmatrix} 0.97412 \to 0.97442 & 0.22469 \to 0.22599 & 0.00337 \to 0.00366 \\ 0.22455 \to 0.22585 & 0.97328 \to 0.97360 & 0.0407 \to 0.0423 \\ 0.00836 \to 0.00896 & 0.0399 \to 0.0415 & 0.999100 \to 0.999167 \end{pmatrix}$$

- The leptonic mixing matrix[4]

$$|U_{\rm PMNS}|_{3\sigma} = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

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- Maybe unification at higher scales e.g GUT scale?
- Use RG equations to obtain values at low scale (M_Z)
- Hierarchical nature of quark masses: Quark mixing angles don't change much (SM/MSSM RG running)

- What about neutrino mixing angles?

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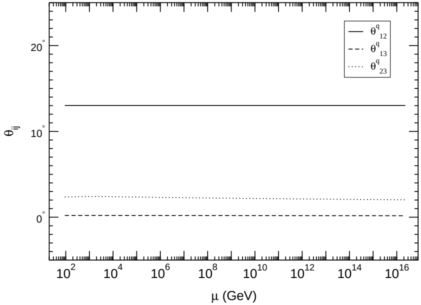

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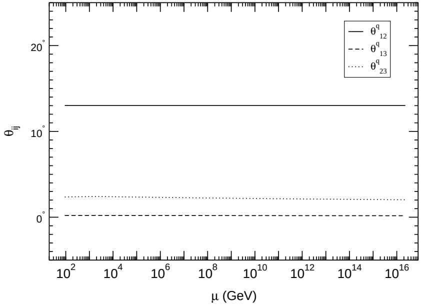

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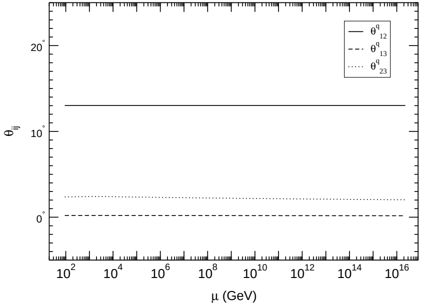

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Radiative Magnification

- High Scale Mixing Unification (HSMU): CKM angles = PMNS angles
- More specifically: For unification at some "High Scale", say GUT

$$\theta_{12}^{0;q} = \theta_{12}^{0} = 13.02°, \quad \theta_{13}^{0;q} = \theta_{13}^{0} = 0.17°, \quad \theta_{23}^{0;q} = \theta_{23}^{0} = 2.03°$$

- Large radiative magnification of PMNS angles is required

$$\theta_{12}^{0} = 13.02° \rightarrow \theta_{12} = 33.36°, \quad \theta_{13}^{0} = 0.17° \rightarrow \theta_{13} = 8.66,$$
$$\theta_{23}^{0} = 2.03° \rightarrow \theta_{23}^{0} = 40.0 \oplus 50.4$$

- Not possible within Standard Model (SM)
- Can be realized within Minimal Supersymmetric Standard Model (MSSM)[5]

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HSMU: General Framework

- Model independent approach: Assume HSMU at some "High Scale"
- Details of the "High Scale" theory not needed
- Below High Scale: MSSM + Type-I seesaw mechanism

$$\mathcal{L}_{MSSM+SSI} = \mathcal{L}_{MSSM} + (Y_\nu)_{ij} \nu^{Ci} \mathbf{h}_a^{(u)} \varepsilon^{ab} \mathbf{l}_b^j \Big|_{\theta\theta} + \frac{1}{2} M_{ij} \nu^{Ci} \nu^{Cj} \Big|_{\theta\theta} + h.c.$$

- Effective left handed neutrino mass matrix

$$m_\nu(\mu) = -\frac{v^2}{2} Y_\nu^T(\mu) M^{-1}(\mu) Y_\nu(\mu)$$

- Right handed neutrinos integrated out below their mass threshold

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$$\mathcal{L}_{MSSM+SSI} = \mathcal{L}_{MSSM} + (Y_\nu)_{ij}\nu^{Ci}\mathbf{h}_a^{(u)}\varepsilon^{ab}\mathbf{l}_b^j\Big|_{\theta\theta} + \frac{1}{2}M_{ij}\nu^{Ci}\nu^{Cj}\Big|_{\theta\theta} + h.c.$$

- Effective left handed neutrino mass matrix

$$m_\nu(\mu) = -\frac{v^2}{2}Y_\nu^T(\mu)M^{-1}(\mu)Y_\nu(\mu)$$

- Right handed neutrinos integrated out below their mass threshold

HSMU: General Framework

- Below seesaw scale: Effective dimension five neutrino mass operator

$$\mathcal{L}_{MSSM+\kappa} = \mathcal{L}_{MSSM} - \frac{1}{4}\kappa_{ij}\mathbf{l}^i_a \varepsilon^{ab}\mathbf{h}^{(u)}_b \mathbf{l}^j_c \varepsilon^{cd}\mathbf{h}^{(u)}_d\Big|_{\theta\theta}$$

- Testing HSMU: Need to run down the masses and mixing parameters from High Scale to low scale (M_Z)
- RG running between High Scale and seesaw scale: Using standard MSSM RG equations within framework of Type-I seesaw mechanism
- Below seesaw scale: RG running with dim-5 operator added to MSSM
- Below SUSY breaking scale: RG running with dim-5 operator added to SM

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RG equations: Mass Matrix

- The RG equation of the effective mass operator[6]

$$16\pi^2 \frac{d\kappa}{dt} = C(Y_e^\dagger Y_e)^T \kappa + C\kappa(Y_e^\dagger Y_e) + \alpha\kappa$$

where $t = \ln(\mu/\mu_0)$, μ is the renormalization scale and $C = 1(\frac{-3}{2})$ in MSSM(SM).

- In MSSM and SM α reads

$$\begin{aligned}\alpha_{\mathrm{MSSM}} &= -\frac{6}{5}g_1^2 - 6g_2^2 + 6(y_t^2 + y_c^2 + y_u^2)\\ \alpha_{\mathrm{SM}} &= -3g_2^2 + 2(y_\tau^2 + y_\mu^2 + y_e^2) + 6(y_t^2 + y_b^2 + y_c^2\\ &+ y_s^2 + y_d^2 + y_u^2) + \lambda\end{aligned}$$

where y_f, $(f = \{e, d, u\})$ are the Yukawa couplings, g_i are gauge couplings and λ is SM Higg's quartic coupling.

[6] S. Antusch, J. Kersten, M. Lindner and M. Ratz, Nucl. Phys. B 674, 401 (2003) [hep-ph/0305273]

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Mass Basis

- Parameters of interest: Masses, mixing angles and physical phases
- Need to go to mass basis: diag(m_1, m_2, m_3)
- Parameterization of PMNS matrix:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{\frac{-i\phi_1}{2}} & 0 & 0 \\ 0 & e^{\frac{-i\phi_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ $(i, j = 1, 2, 3)$

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RG equations: Masses

- RG running of masses[7]

$$16\pi^2 \frac{dm_1}{dt} = \left[\alpha + C y_\tau^2 \left(2 s_{12}^2\, s_{23}^2 + F_1\right)\right] m_1 \ ,$$
$$16\pi^2 \frac{dm_2}{dt} = \left[\alpha + C y_\tau^2 \left(2 c_{12}^2\, s_{23}^2 + F_2\right)\right] m_2 \ ,$$
$$16\pi^2 \frac{dm_3}{dt} = \left[\alpha + 2 C y_\tau^2\, c_{13}^2\, c_{23}^2\right] m_3 \ ,$$

- Where F_1 and F_2 are:

$$F_1 = -s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \cos\delta + 2 s_{13}^2\, c_{12}^2\, c_{23}^2 \ ,$$
$$F_2 = s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \cos\delta + 2 s_{13}^2\, s_{12}^2\, c_{23}^2 \ .$$

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RG equations: Mass Square Differences

- Easily translated into RGEs for the mass squared differences

$$8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\Delta m^2_{\rm sol} = \alpha\,\Delta m^2_{\rm sol} + C y_\tau^2 \left[2 s_{23}^2 \left(m_2^2\, c_{12}^2 - m_1^2\, s_{12}^2\right) + F_{\rm sol}\right] ,$$

$$8\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\Delta m^2_{\rm atm} = \alpha\,\Delta m^2_{\rm atm} + C y_\tau^2 \left[2 m_3^2\, c_{13}^2\, c_{23}^2 - 2 m_2^2\, c_{12}^2\, s_{23}^2 + F_{\rm atm}\right] ,$$

- Where

$$F_{\rm sol} = \left(m_1^2 + m_2^2\right) s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \cos\delta + 2 s_{13}^2\, c_{23}^2 \left(m_2^2\, s_{12}^2 - m_1^2\, c_{12}^2\right) ,$$

$$F_{\rm atm} = -m_2^2\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}\, \cos\delta - 2 m_2^2\, s_{13}^2\, s_{12}^2\, c_{23}^2 .$$

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\end{aligned}
$$

RG equations: Angles

- RG running of angles[8]

$$\dot{\theta}_{12} = -\frac{C y_\tau^2}{32\pi^2} \sin 2\theta_{12}\, s_{23}^2 \frac{|m_1\, e^{i\phi_1} + m_2\, e^{i\phi_2}|^2}{\Delta m^2_{\rm sol}} + \mathbf{O}(\theta_{13})$$

$$\begin{aligned}\dot{\theta}_{13} &= \frac{C y_\tau^2}{32\pi^2} \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{\rm atm}(1+\zeta)} \times \\ &\times \left[m_1 \cos(\phi_1 - \delta) - (1+\zeta)\, m_2 \cos(\phi_2 - \delta) - \zeta m_3 \cos\delta\right] + \mathbf{O}(\theta_{13})\end{aligned}$$

$$\begin{aligned}\dot{\theta}_{23} &= -\frac{C y_\tau^2}{32\pi^2} \sin 2\theta_{23} \frac{1}{\Delta m^2_{\rm atm}} \Big[c_{12}^2\, |m_2\, e^{i\phi_2} + m_3|^2 \\ &+ s_{12}^2 \frac{|m_1\, e^{i\phi_1} + m_3|^2}{1+\zeta}\Big] + \mathbf{O}(\theta_{13})\end{aligned}$$

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RG equations: Dirac Phase

- RG running of Dirac phase[9]

$$\dot{\delta} = \frac{C y_\tau^2}{32\pi^2} \frac{\delta^{(-1)}}{\theta_{13}} + \frac{C y_\tau^2}{8\pi^2} \delta^{(0)} + \mathbf{O}(\theta_{13}) \ ,$$

where

$$\delta^{(-1)} = \sin 2\theta_{12} \sin 2\theta_{23} \frac{m_3}{\Delta m^2_{\mathrm{atm}}\,(1+\zeta)} \times \times \left[m_1 \sin(\phi_1 - \delta) - (1+\zeta)\, m_2 \sin(\phi_2 - \delta) + \zeta m_3 \sin \delta \right] \ ,$$

$$\delta^{(0)} = \frac{m_1 m_2\, s_{23}^2 \sin(\phi_1 - \phi_2)}{\Delta m^2_{\mathrm{sol}}} + m_3\, s_{12}^2 \left[\frac{m_1 \cos 2\theta_{23} \sin \phi_1}{\Delta m^2_{\mathrm{atm}}(1+\zeta)} + \frac{m_2\, c_{23}^2 \sin(2\delta - \phi_2)}{\Delta m^2_{\mathrm{atm}}} \right] + m_3\, c_{12}^2 \left[\frac{m_1\, c_{23}^2 \sin(2\delta - \phi_1)}{\Delta m^2_{\mathrm{atm}}(1+\zeta)} + \frac{m_2 \cos 2\theta_{23} \sin \phi_2}{\Delta m^2_{\mathrm{atm}}} \right] \ .$$

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RG equations: Majorana Phases

- RG running of the physical Majorana phases[10]

$$
\begin{aligned}
\dot{\phi}_1 &= \frac{C y_\tau^2}{4\pi^2} \left\{ m_3 \cos 2\theta_{23} \frac{m_1 s_{12}^2 \sin\phi_1 + (1+\zeta)\, m_2\, c_{12}^2 \sin\phi_2}{\Delta m_{\mathrm{atm}}^2 (1+\zeta)} \right. \\
&\quad \left. + \frac{m_1 m_2\, c_{12}^2\, s_{23}^2 \sin(\phi_1 - \phi_2)}{\Delta m_{\mathrm{sol}}^2} \right\} + \mathbf{O}(\theta_{13}) \,, \\
\dot{\phi}_2 &= \frac{C y_\tau^2}{4\pi^2} \left\{ m_3 \cos 2\theta_{23} \frac{m_1 s_{12}^2 \sin\phi_1 + (1+\zeta)\, m_2\, c_{12}^2 \sin\phi_2}{\Delta m_{\mathrm{atm}}^2 (1+\zeta)} \right. \\
&\quad \left. + \frac{m_1 m_2\, s_{12}^2\, s_{23}^2 \sin(\phi_1 - \phi_2)}{\Delta m_{\mathrm{sol}}^2} \right\} + \mathbf{O}(\theta_{13}) \,.
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HSMU: Assumptions and Initial Conditions

- Natural "High Scale": Grand Unified Theory (GUT) Scale
- Assume HSMU realized at GUT scale i.e. 2×10^{16} GeV
- Sensitivity to choice of high scale: Discussed in later part of talk
- Choice of seesaw scale: HSMU realized for varied range of seesaw scale
- For sake of definiteness: Choose typical Seesaw scale of 10^{12} GeV
- SUSY breaking scale: 5 TeV
- Dependence on choice of SUSY breaking scale: Discussed in later part of talk
- Larger values of $\tan\beta$: Enhanced magnification
- We choose $\tan\beta = 55$: Dependence on $\tan\beta$ discussed in later part of talk

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- Large radiative magnification: Quasi degenerate neutrinos with normal hierarchy
- Inverted Hierarchy: No magnification of θ_{23}
- Assume no CP violation in leptonic sector: Dirac as well as Majorana phases taken zero
- CP violating scenario: Effect of phases on HSMU discussed in later part of talk

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Implementing HSMU: Two step process

- **Bottom - Up**
- Start from known values of gauge couplings, quark mixing angles, masses of quarks and charged leptons at low energies (M_Z)
- Use RG equations: Obtain the corresponding values at high energies
- HSMU hypothesis: Take neutrino mixing angles same as the quark mixing angles at the unification scale
- **Top - Down**
- Neutrino masses at high scale: Unknown parameters
- Determine these three parameters such that: Low energy values of the oscillation parameters i.e. $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}, \theta_{23}$ and θ_{13} agree with their present experimental ranges

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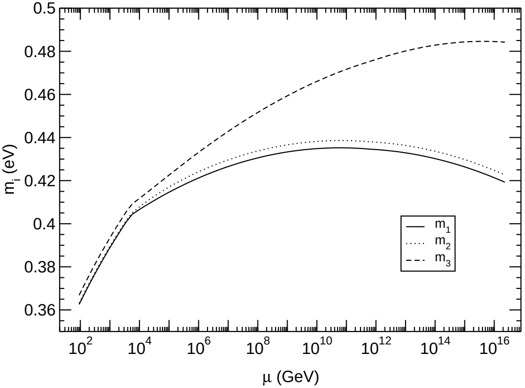
The RG evolution of neutrino masses m_i



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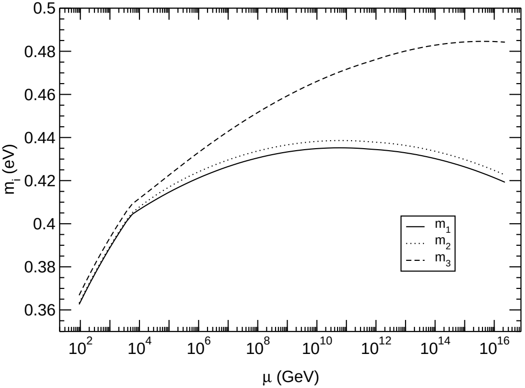

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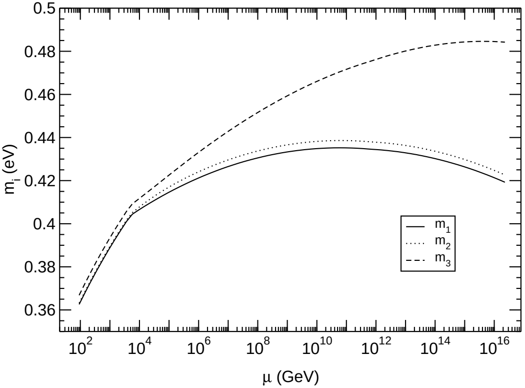

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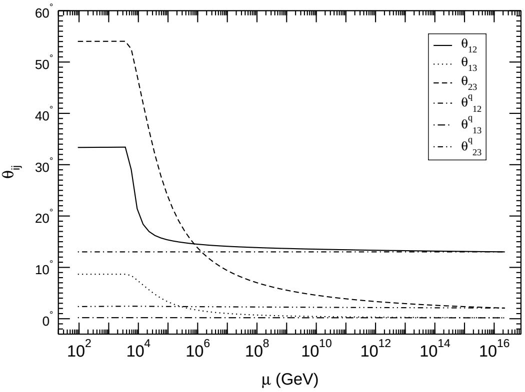

- Hierarchical quark masses: RG running in quark sector is almost negligible
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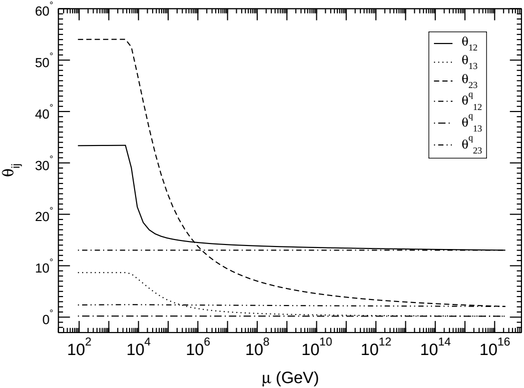

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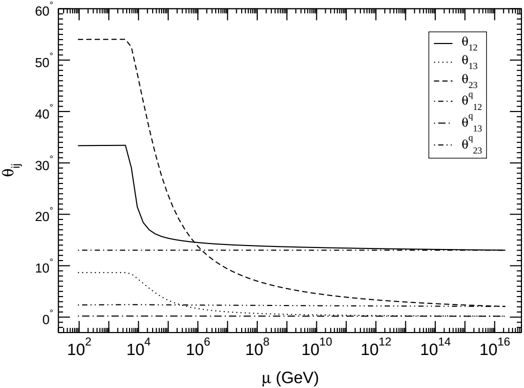

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Numerical results on the evolution of masses and mixing

	I	II	III
m_1^0(eV)	0.4196	0.4146	0.4286
m_2^0(eV)	0.4230	0.4180	0.4320
m_3^0(eV)	0.4843	0.4786	0.4946
m_1(eV)	0.3626	0.3582	0.3703
m_2(eV)	0.3632	0.3589	0.3709
m_3(eV)	0.3668	0.3625	0.3746
$\Delta m^2_{21}(eV^2)_{RG}$	4.30×10^{-4}	4.49×10^{-4}	4.20×10^{-4}
$\Delta m^2_{32}(eV^2)_{RG}$	2.67×10^{-3}	2.62×10^{-3}	2.78×10^{-3}
$M_{\tilde{e}}/M_{\tilde{\mu},\tilde{\tau}}$	1.94	1.84	2.16
$\Delta m^2_{21}(eV^2)_{th}$	-3.55×10^{-4}	-3.73×10^{-4}	-3.44×10^{-4}
$\Delta m^2_{32}(eV^2)_{th}$	-2.74×10^{-4}	-2.16×10^{-4}	-3.82×10^{-4}
$\Delta m^2_{21}(eV^2)_{tot}$	7.50×10^{-5}	7.58×10^{-5}	7.59×10^{-5}
$\Delta m^2_{32}(eV^2)_{tot}$	2.40×10^{-3}	2.40×10^{-3}	2.40×10^{-3}
$\theta_{23}/^\circ$	54.03	53.93	54.18
$\theta_{13}/^\circ$	8.66	8.67	8.67
$\theta_{12}/^\circ$	33.36	31.14	35.87

Testing HSMU: Values of $m_{\beta\beta}$ and m_β

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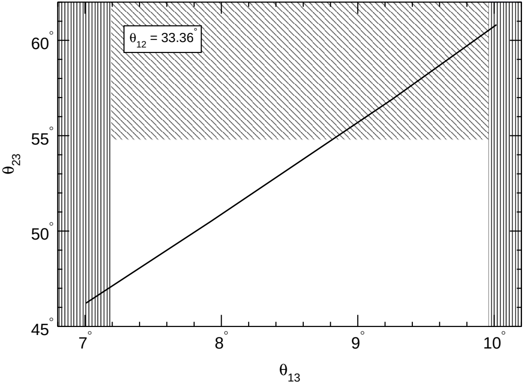

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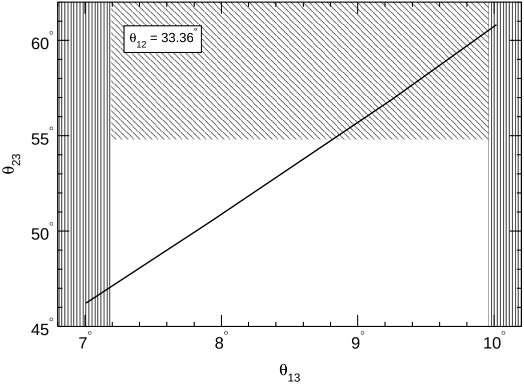

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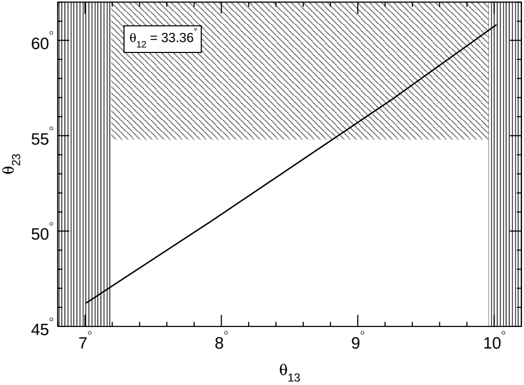

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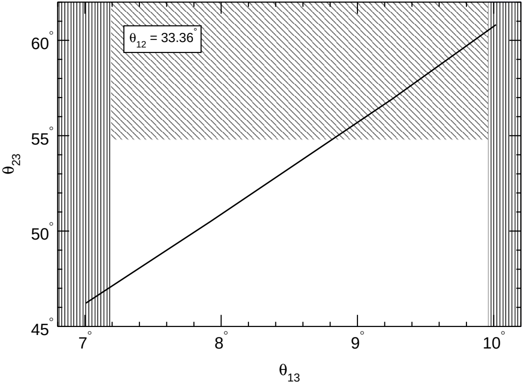

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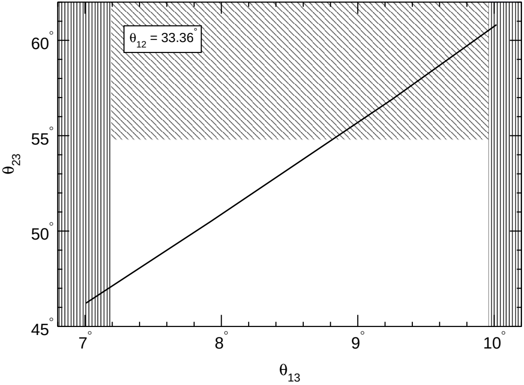

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- HSMU realized for Majorana neutrinos: Requires quasi-degeneracy and normal hierarchy
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Outline

Nature of Neutrinos: Dirac or Majorana?

- One of the most important open questions in neutrino physics: Whether neutrinos are Dirac or Majorana particles
- Answering this question: Essential to find the underlying theory of neutrino masses and mixing.
- Current understanding: Dirac neutrinos as plausible as Majorana ones
- Neutrinoless double beta decay experiments: Dedicated ongoing experiments to determine the nature of neutrinos.
- No conclusive evidence: Neutrinoless double beta decay experiments have not seen any signal so far[12].
- Instructive to see if HSMU can be implemented for Dirac Neutrinos as well

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RG equation: Masses

- RG running of masses[13]

$$16\pi^2 \dot{m}_1 = \Big\{ C\, y_\tau^2 \Big[\cos^2\theta_{12}\, \cos^2\theta_{23}\, \sin^2\theta_{13} + \sin^2\theta_{12}\, \sin^2\theta_{23} - \frac{1}{2}\cos\delta\, \sin\theta_{13}\, \sin(2\,\theta_{12})\, \sin(2\,\theta_{23}) \Big] + \alpha \Big\}\, m_1 \,,$$

$$16\pi^2 \dot{m}_2 = \Big\{ C\, y_\tau^2 \Big[\sin^2\theta_{12}\, \cos^2\theta_{23}\, \sin^2\theta_{13} + \cos^2\theta_{12}\, \sin^2\theta_{23} + \frac{1}{2}\cos\delta\, \sin\theta_{13}\, \sin(2\,\theta_{12})\, \sin(2\,\theta_{23}) \Big] + \alpha \Big\}\, m_2 \,,$$

$$16\pi^2 \dot{m}_3 = \{ C\, y_\tau^2\, \cos^2\theta_{13}\, \cos^2\theta_{23} + \alpha \}\, m_3 \,.$$

[13] M. Lindner, M. Ratz and M. A. Schmidt, JHEP 0509, 081 (2005), hep-ph/0506280

RG equation: Angles

- RG running of mixing angles[14]

$$\dot{\theta}_{12} = \frac{-C\, y_\tau^2}{32\,\pi^2}\,\frac{m_1^2+m_2^2}{m_2^2-m_1^2}\,\sin(2\,\theta_{12})\,\sin^2\theta_{23} + \mathbf{O}(\theta_{13})\,,$$

$$\dot{\theta}_{13} = \frac{-C\, y_\tau^2}{32\,\pi^2}\,\frac{1}{(m_3^2-m_1^2)\,(m_3^2-m_2^2)}\Big\{\big(m_2^2-m_1^2\big)\; m_3^2\; \cos\delta\,\cos\theta_{13}\,\sin(2\,\theta_{12})\,\sin(2\,\theta_{23}) + \big[m_3^4 - \big(m_2^2-m_1^2\big)\, m_3^2\,\cos(2\,\theta_{12}) - m_1^2\,m_2^2\big]\,\cos^2\theta_{23}\,\sin(2\,\theta_{13})\Big\}\,,$$

$$\dot{\theta}_{23} = \frac{-C\, y_\tau^2}{32\,\pi^2}\,\frac{\big[m_3^4 - m_1^2\,m_2^2 + (m_2^2-m_1^2)\,m_3^2\,\cos(2\,\theta_{12})\big]}{(m_3^2-m_1^2)\,(m_3^2-m_2^2)}\,\sin(2\,\theta_{23}) + \mathbf{O}(\theta_{13})\,,$$

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RG equation: Dirac Phase

- RG running of Dirac phase[15]

$$\dot{\delta} = \dot{\delta}^{(-1)}\theta_{13}^{-1} + \dot{\delta}^{(0)} + \dot{\delta}^{(1)} + \mathbf{O}\left(\theta_{13}^2\right) ,$$

where

$$\dot{\delta}^{(-1)} = \frac{C\, y_\tau^2}{32\,\pi^2}\,\frac{(m_2^2 - m_1^2)\, m_3^2}{(m_3^2 - m_1^2)\,(m_3^2 - m_2^2)}\,\sin(\delta)\,\sin(2\,\theta_{12})\,\sin(2\,\theta_{23}) ,$$

$$\dot{\delta}^{(0)} = 0$$

$$\dot{\delta}^{(1)} = \frac{C\, y_\tau^2}{16\,\pi^2}\,\frac{m_2^2\,\left(m_3^2 - m_1^2\right)^2}{(m_2^2 - m_1^2)\,(m_3^2 - m_1^2)\,(m_3^2 - m_2^2)}\,\cot(\theta_{12})\,\sin(2\,\theta_{23})\,\sin\delta + \ldots ,$$

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Implementing HSMU

- Same as before
- Choose: Unification scale $= 2 \times 10^{16}$ GeV, SUSY breaking scale $= 5$ TeV and $\tan\beta = 55$
- **Bottom - Up**
- Start from known values of gauge couplings, quark mixing angles, masses of quarks and charged leptons at low energies (M_Z)
- Use RG equations: Obtain the corresponding values at high energies
- HSMU hypothesis: Take neutrino mixing angles and phase same as the quark mixing angles and phase at the unification scale
- **Top - Down**
- Neutrino masses at high scale: Unknown parameters
- Determine these three parameters such that: Low energy values of the oscillation parameters i.e. $\Delta m^2_{12}, \Delta m^2_{23}, \theta_{12}, \theta_{23}$ and θ_{13} agrees with their present experimental ranges

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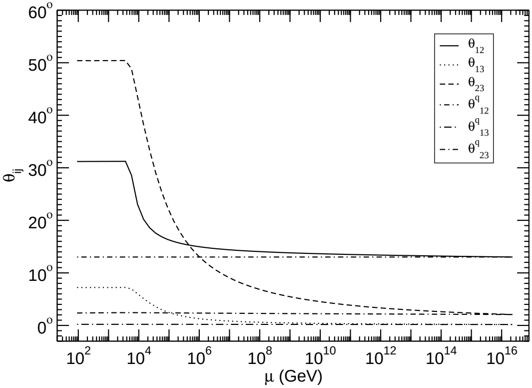
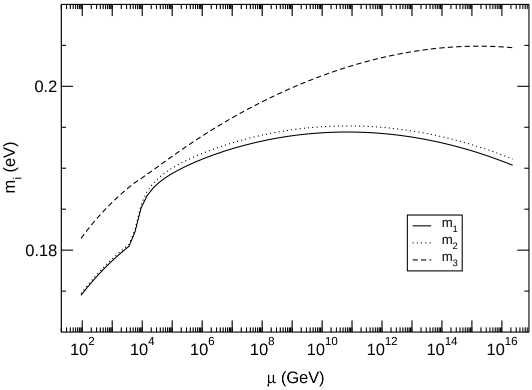
The RG evolution of neutrino mixing angles θ_{ij}

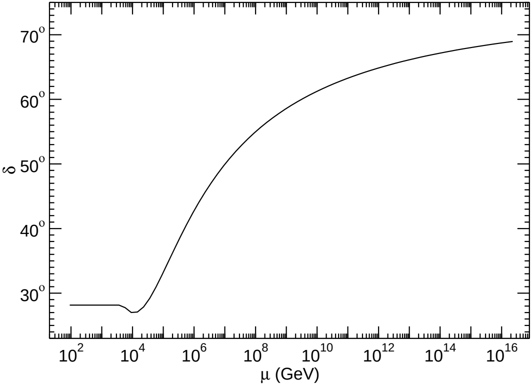


- Quasi-degenerate neutrinos: Large angle magnification in leptonic sector

The RG evolution of neutrino masses m_i



The RG evolution of Dirac Phase



Numerical results

- Bottom-up running:
  $\theta_{12}^{0,q} = 13.02^\circ$, $\theta_{13}^{0,q} = 0.17^\circ$, $\theta_{23}^{0,q} = 2.03^\circ$ and $\delta_{CP}^{0,q} = 68.93^\circ$.
- Following HSMU, the neutrino mixing parameters at unification scale are taken to be same as those of quark mixing parameters.
- At unification scale, we choose:
  $m_2^0 = 0.1912\ eV$, $\Delta m_{21}^2 = 2.8478 \times 10^{-4}\ eV^2$,
  $\Delta m_{32}^2 = 5.3602 \times 10^{-3}\ eV^2$.
- Top-down running:
  $\theta_{12} = 31.20^\circ$, $\theta_{13} = 7.22^\circ$, $\theta_{23} = 50.39^\circ$, $\delta_{CP} = 28.14^\circ$,
  $J_{\rm CP} = 0.102$, $m_2 = 0.1747$ eV, $\Delta m_{sol}^2 = 7.750 \times 10^{-5}\ eV^2$,
  $\Delta m_{atm}^2 = 2.399 \times 10^{-3}\ eV^2$.
- All low scale parameters are within their 3-σ range.
- Threshold corrections are NOT required.
- The mean mass $m = 0.1769$ eV and the "averaged electron neutrino mass" $m_\beta = 0.1747$ eV (slightly below the present reach of KATRIN experiment).

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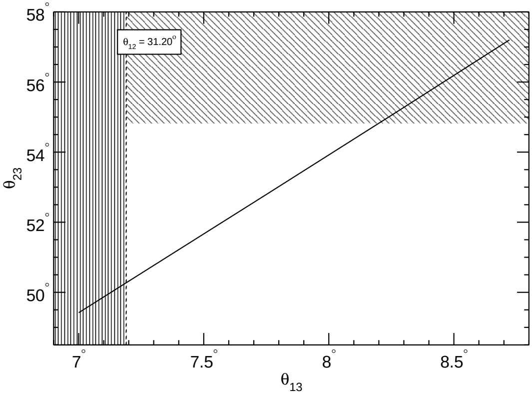
  $\theta_{12} = 31.20^\circ$, $\theta_{13} = 7.22^\circ$, $\theta_{23} = 50.39^\circ$, $\delta_{CP} = 28.14^\circ$,
  $J_{\rm CP} = 0.102$, $m_2 = 0.1747$ eV, $\Delta m_{sol}^2 = 7.750 \times 10^{-5}\ eV^2$,
  $\Delta m_{atm}^2 = 2.399 \times 10^{-3}\ eV^2$.
- All low scale parameters are within their 3-σ range.
- Threshold corrections are NOT required.
- The mean mass $m = 0.1769$ eV and the "averaged electron neutrino mass" $m_\beta = 0.1747$ eV (slightly below the present reach of KATRIN experiment).

Octant of θ_{23}



- Correlated RG evolution of θ_{13} and θ_{23}: θ_{23} non maximal and in second octant

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- HSMU can be realized for both Dirac as well as Majorana neutrinos
- Several important predictions for Dirac case:
  - Dirac nature: No neutrinoless double beta decay
  - "Averaged electron neutrino mass" m_β: Slightly below KATRIN's proposed sensitivity
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Outline

Variation of HSMU and SUSY Breaking Scale

- So far we assumed HSMU to be realized at GUT scale
- HSMU does not depend on "details" of GUT scale theory
- Instructive to analyze the effect of variation of HSMU scale
- Similarly SUSY breaking scale and tan β were taken as 5 TeV and 55, respectively
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Scale of HSMU
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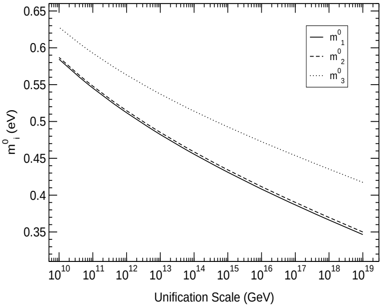

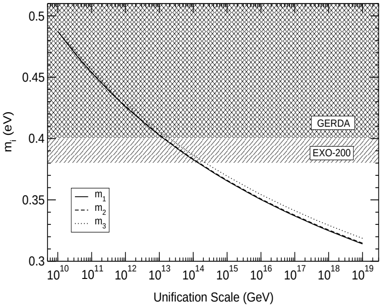

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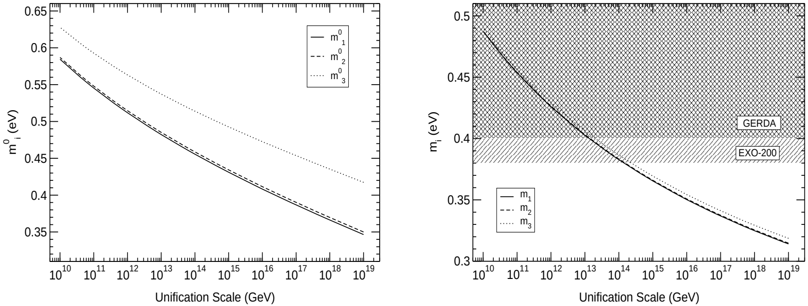

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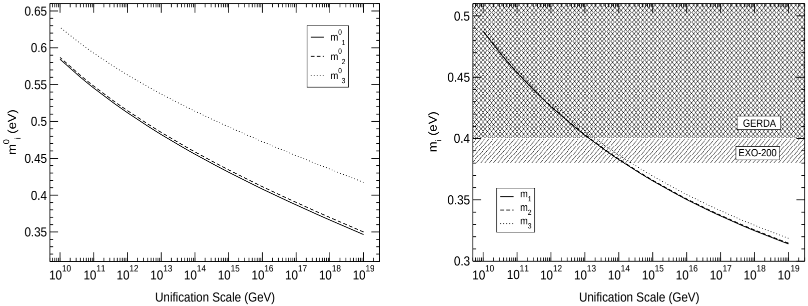

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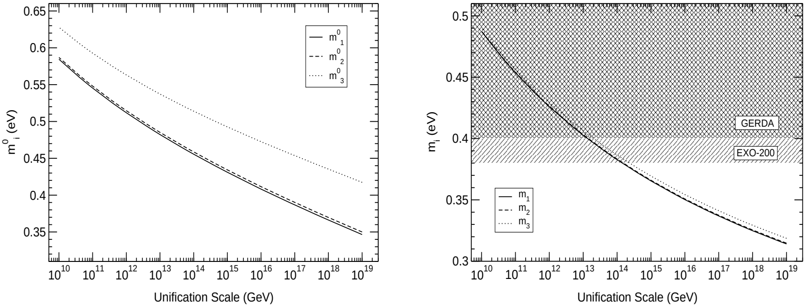

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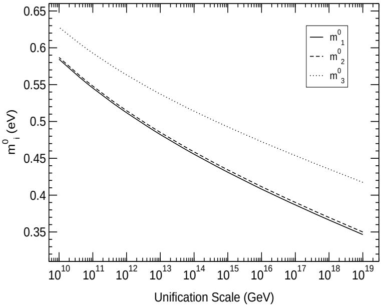

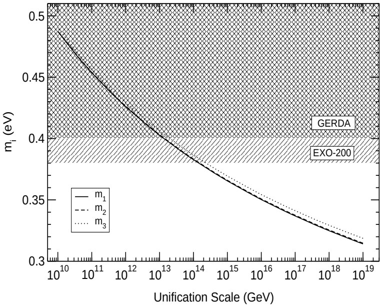

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SUSY Breaking Scale

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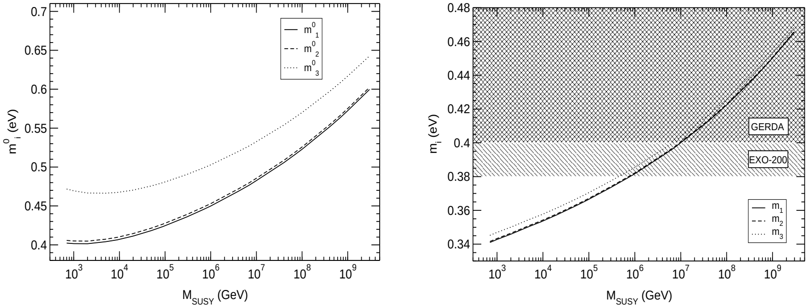

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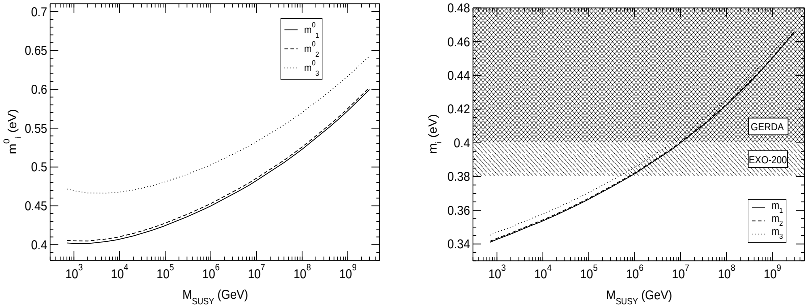

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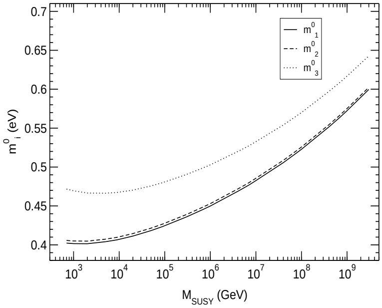

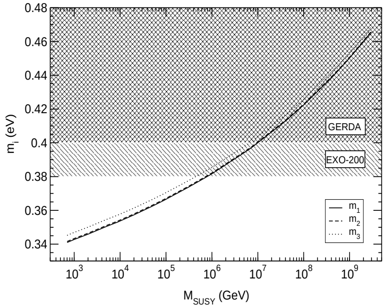

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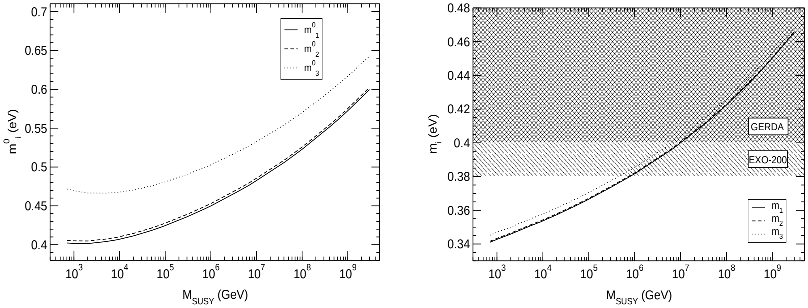

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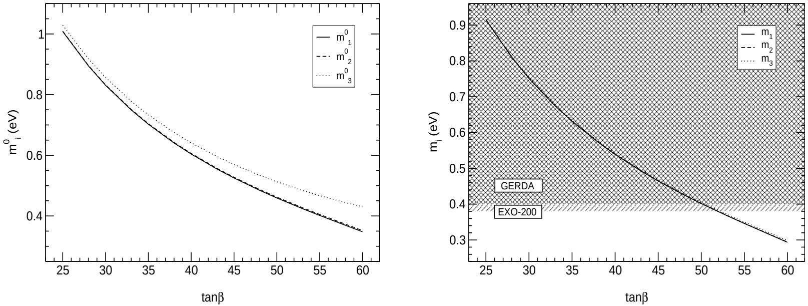

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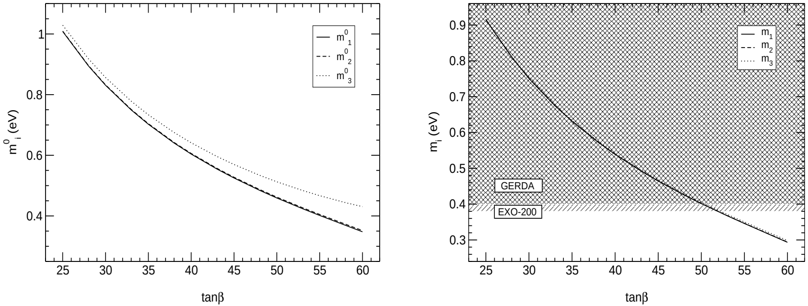
Choice of tan β

Majorana Neutrinos



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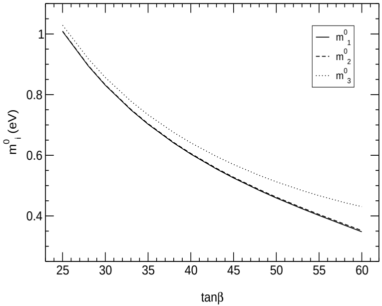

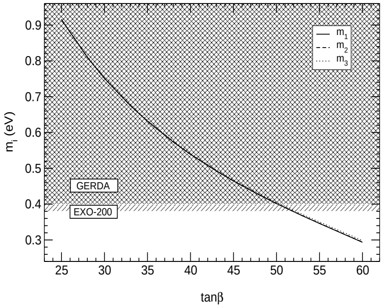

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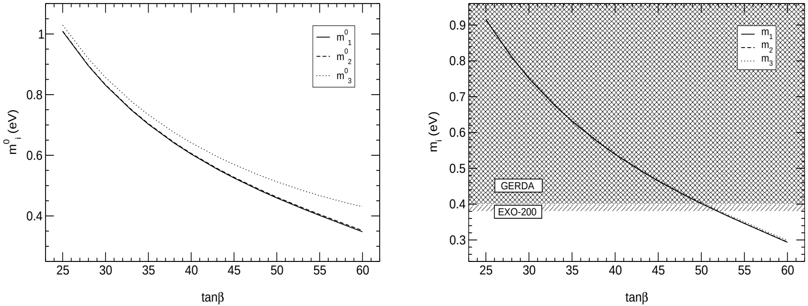

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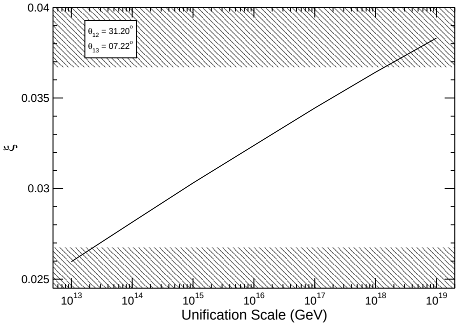

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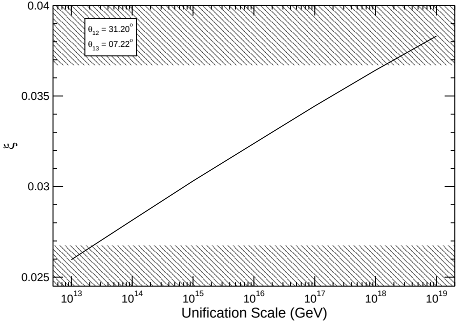

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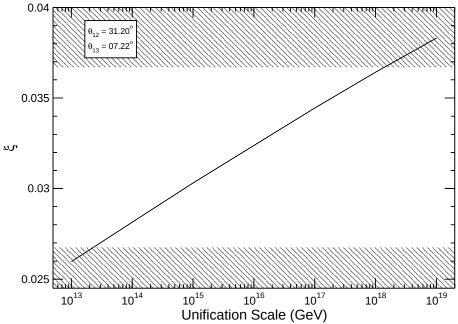

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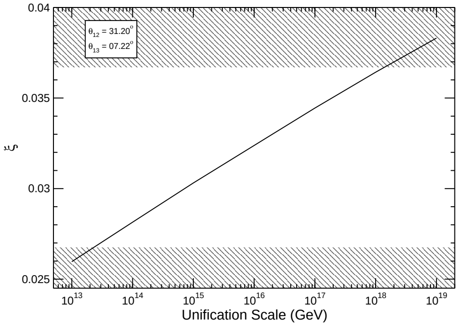

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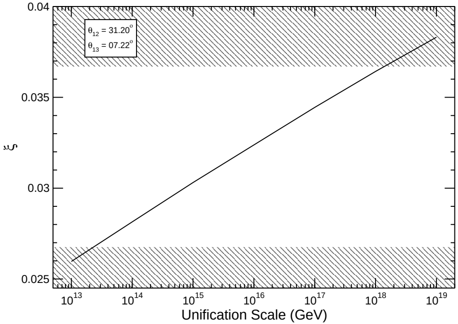

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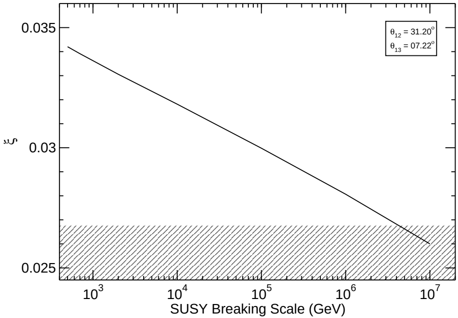

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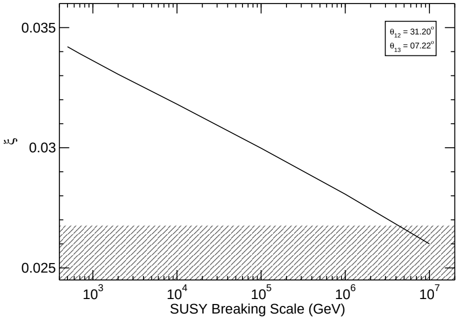

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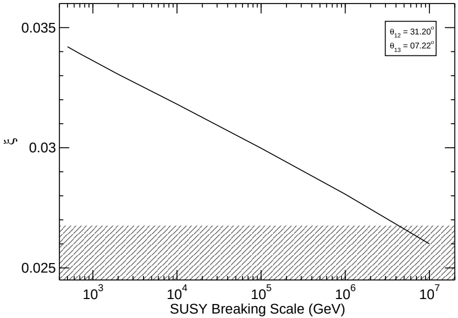

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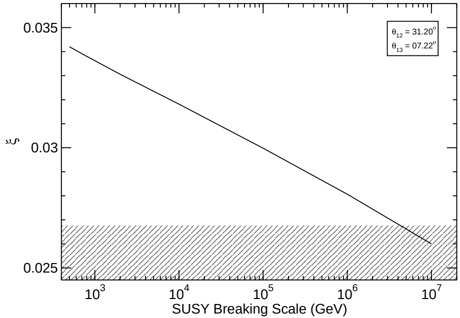

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- This assumption need not be realized in nature
- The Dirac and Majorana phases (for Majorana neutrinos) enter RG equations of all other parameters
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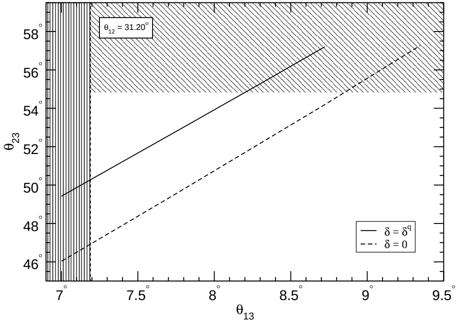

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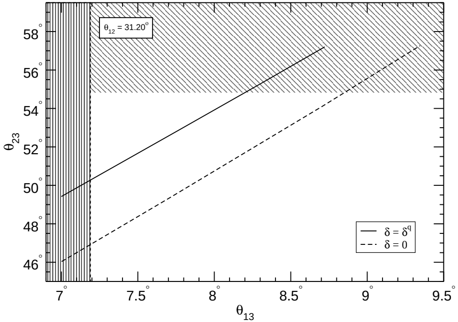

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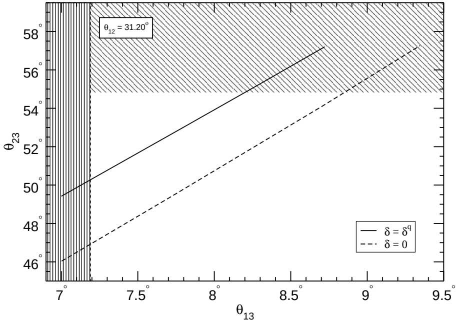

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Case I: $\delta_{\rm CP} = \delta^{q}_{\rm CP}$, $\phi_1 = \phi_2 = 0$

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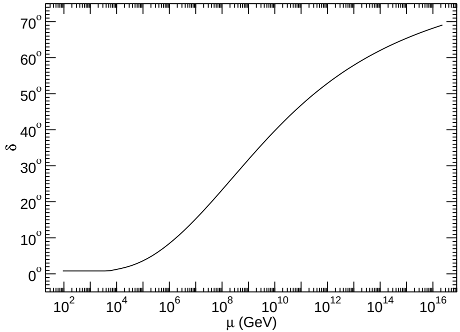

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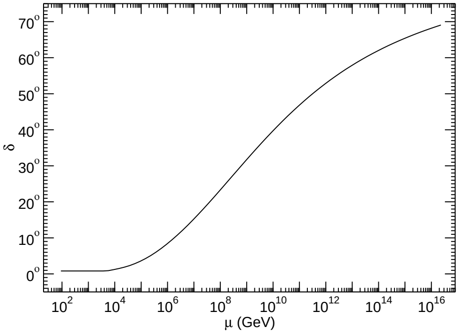

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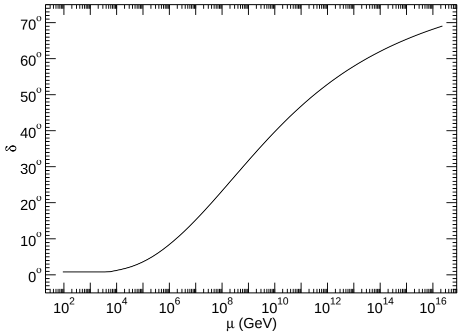

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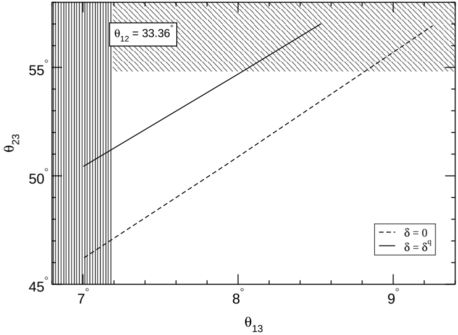

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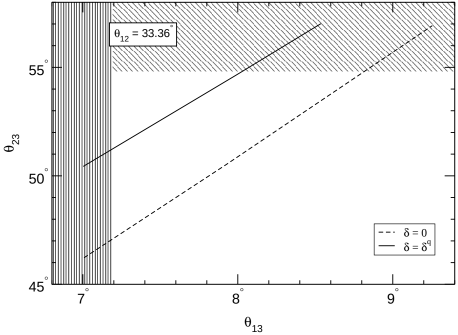

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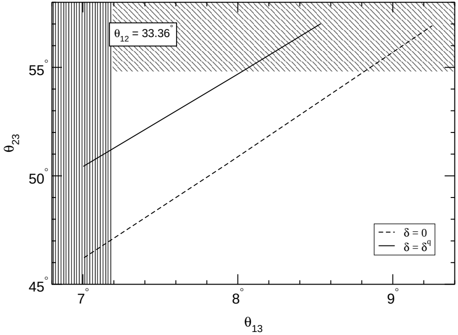

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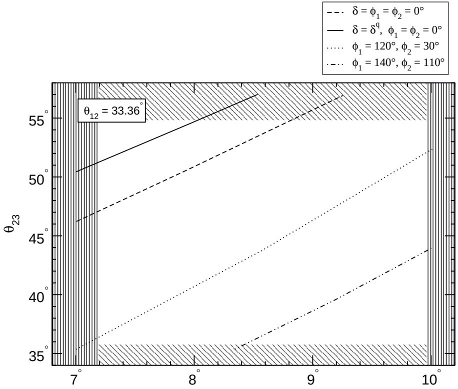

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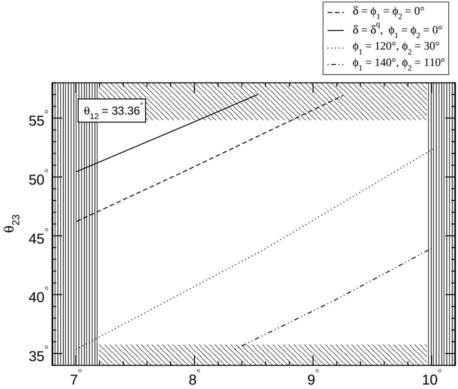

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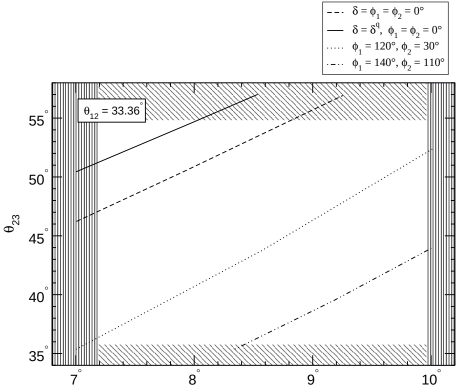

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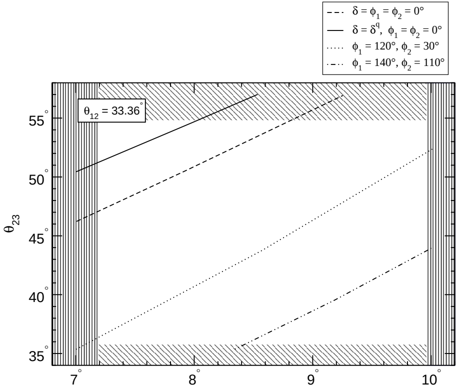

Outline

Testing HSMU Hypothesis

- HSMU is quite predictive
- Several experiments can test its predictions
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Experiment	Majorana	Dirac
$m_{\beta\beta}$ (observed)	✓	×
$m_{\beta\beta} < 0.1$ eV	×	✓
m_β (observed) KATRIN	✓	✓
m_β (not observed) KATRIN	×	✓
$\theta_{23} > 45^\circ$	✓	✓
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Mass Hierarchy (Inverted)	×	×

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Conclusions and Future Work

- High Scale Mixing Unification (HSMU) of PMNS and CKM parameters is an interesting possibility
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- It naturally leads to non zero and “relatively large” values of θ_{13} consistent with present global fits
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