Two Texture Zeros and Near Maximal Atmospheric Neutrino Mixing Angle

Radha Raman Gautam

Department of Physics, Panjab University, Chandigarh-160014, INDIA
In collaboration with: M. M. Gupta, S. Dev, L. Singh

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Outline

- The Neutrino Mass Matrix
- Two Texture Zeros
- Symmetry Realization
- Stability of Texture Zeros
- Summary

The Neutrino Mass Matrix

- For Majorana neutrinos the neutrino mass matrix is complex symmetric and can be diagonalized as:

$$M_\nu = V M_\nu^{diag} V^T \quad \text{where} \quad M_\nu^{diag} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

- We can write the matrix V as $V = UP$ where

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}$ is the diagonal phase matrix with the Majorana-type CP-violating phases α, β and the Dirac-type CP- violating phase δ.

- The matrix V is the neutrino mixing matrix.

The Neutrino Mass Matrix

- The mass matrix for Majorana neutrinos contains nine physical parameters Viz. the three neutrino masses, three mixing angles and the three CP-Violating phases (one Dirac phase and two Majorana phases).
- The two mass-squared differences (Δm^2_{21} and $|\Delta m^2_{31}|$) and three mixing angles (solar θ_{12}, atmospheric θ_{23} and reactor θ_{13}) have been measured in Solar, Atmospheric and Reactor experiments.
- The mixing angle θ_{13} was measured recently measured with a good precision[1] and its best fit is around '9°'.
- The CP violating phases are unconstrained at present.

[1] K. Abe *et al.* [T2K collaboration], *Phys. Rev. Lett.* **107**, 041801 (2011), arXiv:1106.2822 [hep-ex]; P. Adamson *et al.* [MINOS collaboration], *Phys. Rev. Lett.* **107**, 181802 (2011), arXiv:1108.0015 [hep-ex]; Y. Abe *et al.*, [Double Chooz collaboration], *Phys. Rev. Lett.* **108**, 131801 (2012), arXiv:1112.6353 [hep-ex]; F. P. An *et al.*, [Daya Bay collaboration], *Phys. Rev. Lett.* **108**, 171803 (2012), arXiv:1203.1669 [hep-ex]; Soo-Bong Kim, for RENO collaboration, *Phys. Rev. Lett.* **108**, 191802 (2012), arXiv:1204.0626 [hep-ex].

Neutrino Oscillation Data

Parameter	mean $^{(+1\sigma,+2\sigma,+3\sigma)}_{(-1\sigma,-2\sigma,-3\sigma)}$
$\Delta m^2_{21}[10^{-5}eV^2]$	$7.62^{(+0.19,+0.39,+0.58)}_{(-0.19,-0.35,-0.5)}$
$\Delta m^2_{31}[10^{-3}eV^2]$	$2.55^{(+0.06,+0.13,+0.19)}_{(-0.09,-0.19,-0.24)}$, $(-2.43^{(+0.09,+0.19,+0.24)}_{(-0.07,-0.15,-0.21)})$
$\sin^2\theta_{12}$	$0.32^{(+0.016,+0.03,+0.05)}_{(-0.017,-0.03,-0.05)}$
$\sin^2\theta_{23}$	$0.613^{(+0.022,+0.047,+0.067)}_{(-0.04,-0.233,-0.25)}$, $(0.60^{(+0.026,+0.05,+0.07)}_{(-0.031,-0.210,-0.230)})$
$\sin^2\theta_{13}$	$0.0246^{(+0.0028,+0.0056,+0.0076)}_{(-0.0029,-0.0054,-0.0084)}$, $(0.0250^{(+0.0026,+0.005,+0.008)}_{(-0.0027,-0.005,-0.008)})$

Table: Current Neutrino oscillation parameters from global fits.[3] The upper (lower) row corresponds to Normal (Inverted) Spectrum, with $\Delta m^2_{31} > 0$ ($\Delta m^2_{31} < 0$).

[3] M. Tortola, J. W. F. Valle and D. Vanegas, *Phys. Rev.* **D 86**, 073012 (2012), arXiv:1205.4018 [hep-ph].

Neutrino Masses

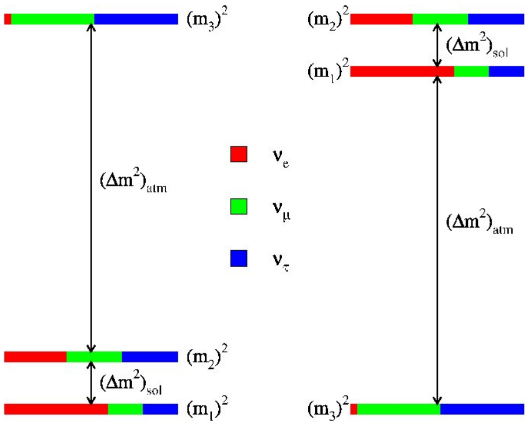

Figure: [5]Neutrino mass and flavor spectra for the normal and inverted mass hierarchies.

[5] J. L. Hewett, et al. arXiv:1205.2671 [hep-ex]

Neutrino Masses

- The effective Majorana mass of the electron neutrino ($|M_{ee}|$) which determines the rate of neutrinoless double beta (NDB) decay is given by

$$|M_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}|.$$

- The possible measurement of the effective Majorana mass in neutrinoless double beta decay searches will provide additional constraint on the three neutrino parameters viz. the neutrino mass scale and the two Majorana-type CP violation phases.
- But the two Majorana phases will not be uniquely determined from the effective Majorana Mass measurements even if the neutrino mass scale is known.
- Neutrino mass scale may be independently determined by the direct beta decay searches and cosmological observations.

Neutrino Masses

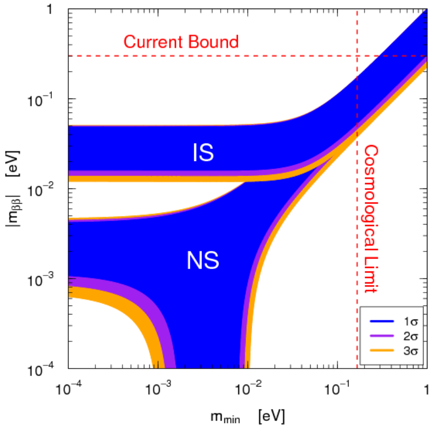

Figure: [6]Range for $|M_{ee}|$ as a function of smallest neutrino mass.

[6]S. M. Bilenky, C. Giunti, *Mod. Phys. Lett.* **A 27** 12300157, arXiv:1203.5250 [hep-ph]

The Neutrino Mass Matrix

The neutrino mass matrix contains more unknown parameters than the results of the foreseeable experiments.

Neutrino Mass Matrix is not well determined

The strategy is to employ other theoretical inputs to reduce the number of parameters in the neutrino mass matrix.

Neutrino Masses and Mixings

There are mainly two approaches to explain neutrino masses and mixings:

1. Mass independent textures (or form diagonalizable textures) which lead to mixing matrices independent of the eigenvalues.
2. Mass dependent textures which induce relations between mixing matrix elements and masses.

- The Most studied example of mass independent textures is Tribimaximal mixing(TBM) which predicts a vanishing 1-3 mixing angle $\theta_{13} = 0$, maximal 2-3 mixing angle $\theta_{23} = \pi/4$ and 1-2 mixing angle $\theta_{12} = \sin^{-1}(1/\sqrt{3})$.
- Some examples of mass dependent textures are texture zeros, vanishing minors, hybrid textures.

Two Texture Zeros

A_1	A_2	B_1
$\begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix}$	$\begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$	$\begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix}$
B_2	B_3	B_4
$\begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$	$\begin{pmatrix} X & 0 & X \\ 0 & 0 & X \\ X & X & X \end{pmatrix}$	$\begin{pmatrix} X & X & 0 \\ X & X & X \\ 0 & X & 0 \end{pmatrix}$
C	-	-
$\begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & 0 \end{pmatrix}$	-	-

Table: Viable two zero texture neutrino mass matrices. X denote the non zero elements.

Two Texture Zeros

- The two texture zeros at (p, q) and (r, s) positions in the neutrino mass matrix give two complex equations viz.

$$m_1 U_{p1} U_{q1} + e^{2i\alpha} m_2 U_{p2} U_{q2} + e^{2i(\beta+\delta)} m_3 U_{p3} U_{q3} = 0$$

$$m_1 U_{r1} U_{s1} + e^{2i\alpha} m_2 U_{r2} U_{s2} + e^{2i(\beta+\delta)} m_3 U_{r3} U_{s3} = 0$$

where p, q, r and s can take the values e, μ and τ.

- Solving the above two equations simultaneously, we obtain

$$\frac{m_1}{m_2} e^{-2i\alpha} = \frac{U_{r2} U_{s2} U_{p3} U_{q3} - U_{p2} U_{q2} U_{r3} U_{s3}}{U_{p1} U_{q1} U_{r3} U_{s3} - U_{p3} U_{q3} U_{r1} U_{s1}},$$

$$\frac{m_1}{m_3} e^{-2i\beta} = \frac{U_{r3} U_{s3} U_{p2} U_{q2} - U_{p3} U_{q3} U_{r2} U_{s2}}{U_{p1} U_{q1} U_{r2} U_{s2} - U_{p2} U_{q2} U_{r1} U_{s1}} e^{2i\delta}.$$

- The CP-violating Majorana phases α and β are given by

$$\alpha = -\frac{1}{2} \mathrm{Arg} \left(\frac{U_{r2} U_{s2} U_{p3} U_{q3} - U_{p2} U_{q2} U_{r3} U_{s3}}{U_{p1} U_{q1} U_{r3} U_{s3} - U_{p3} U_{q3} U_{r1} U_{s1}} \right),$$

$$\beta = -\frac{1}{2} \mathrm{Arg} \left(\frac{U_{r3} U_{s3} U_{p2} U_{q2} - U_{p3} U_{q3} U_{r2} U_{s2}}{U_{p1} U_{q1} U_{r2} U_{s2} - U_{p2} U_{q2} U_{r1} U_{s1}} e^{2i\delta} \right).$$

Two Texture Zeros

- The two mass ratios can be further used to obtain two values of m_1 viz.

$$m_1 = \eta\sqrt{\frac{\Delta m_{21}^2}{1-\eta^2}}\ , \quad m_1 = \rho\sqrt{\frac{\Delta m_{21}^2 + |\Delta m_{23}^2|}{1-\rho^2}}$$

where $(\Delta m_{ij}^2 \equiv m_i^2 - m_j^2)$, $\eta = \left|\frac{m_1}{m_2}e^{-2i\alpha}\right|$ and $\rho = \left|\frac{m_1}{m_3}e^{-2i\beta}\right|$.

- The above two values of m_1 contain the constraints of two texture zeros in M_ν through the two mass ratios η and ρ.
- The simultaneous existence of two texture zeros in M_ν requires these two values of m_1 to be equal.

Two Texture Zeros

- In the numerical analysis, firstly, we use the experimental input of the two mass squared differences $(\Delta m^2_{21}, \Delta m^2_{23})$ along with the constraints of two texture zeros and large $|M_{ee}|$ to obtain predictions for mixing angles.
- We vary the two mass squared differences randomly within the 3σ allowed ranges but keep the neutrino mixing angles free and vary them between 0° and 90°.
- The Dirac phase is varied from 0° to 360° and the constraint of a large $|M_{ee}| > 0.08$ eV is imposed.
- We carry out this analysis for classes B_1, B_2, B_3 and B_4.
- In the second step, we also take into account the experimental input of mixing angles.

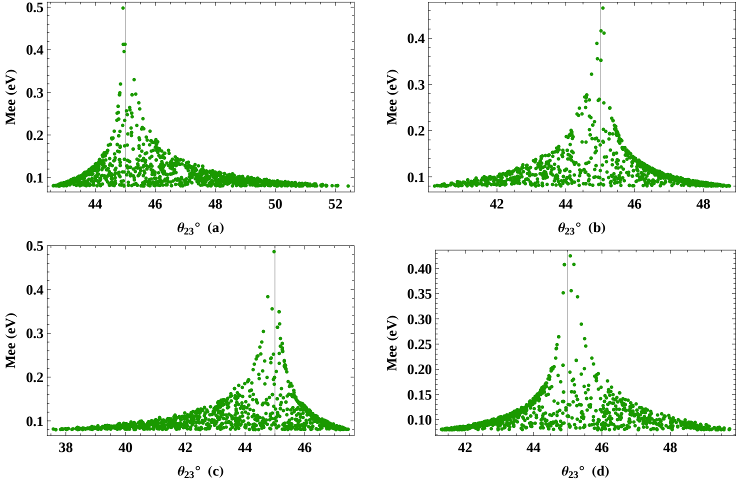

Figure: Correlation plots for classes B_1(NH)(a), B_1(IH)(b), B_2(NH)(c) and B_2(IH)(d). Here the mixing angles are varied between 0° and 90°.[8]

[8] S. Dev, R. R. Gautam, L. Singh, M. M. Gupta, arXiv:1405.0566 [hep-ph].

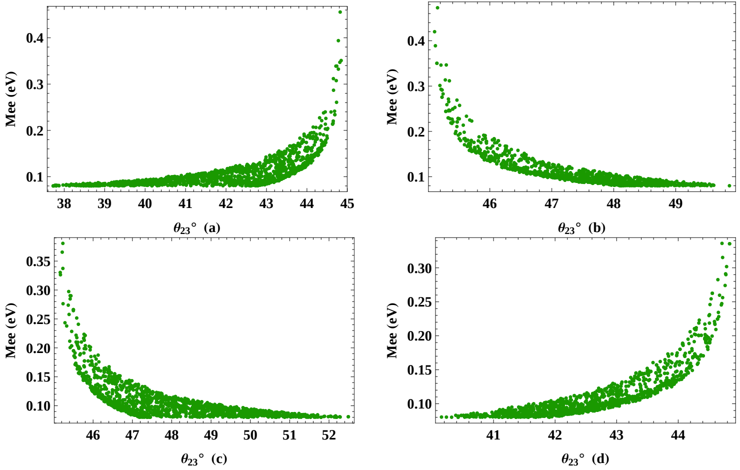

Figure: Correlation plots for classes B_3(NH)(a), B_3(IH)(b), B_4(NH)(c) and B_4(IH)(d). Here the mixing angles are varied between 0° and 90°.[10]

[10] S. Dev, R. R. Gautam, L. Singh, M. M. Gupta, arXiv:1405.0566 [hep-ph].

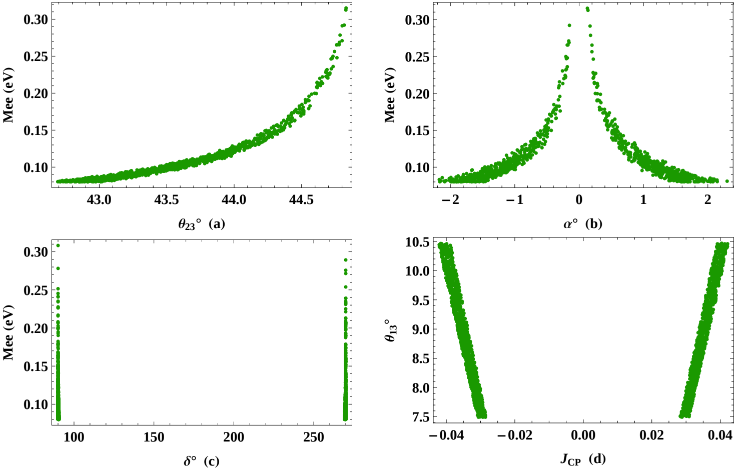

Figure: Correlation plots for classes B_1(NH)(a, b) and B_3(IH)(c, d).[11]

[11] S. Dev, R. R. Gautam, L. Singh, M. M. Gupta, arXiv:1405.0566 [hep-ph].

Class	M_ν
B_1	$\begin{pmatrix} 0.1 & \lambda^2 & 0 \\ \lambda^2 & 0 & 0.1 \\ 0 & 0.1 & \lambda \end{pmatrix}$
B_2	$\begin{pmatrix} 0.1 & 0 & \lambda^2 \\ 0 & \lambda & 0.1 \\ \lambda^2 & 0.1 & 0 \end{pmatrix}$
B_3	$\begin{pmatrix} 0.1 & 0 & \lambda^2 \\ 0 & 0 & 0.1 \\ \lambda^2 & 0.1 & \lambda \end{pmatrix}$
B_4	$\begin{pmatrix} 0.1 & \lambda^2 & 0 \\ \lambda^2 & \lambda & 0.1 \\ 0 & 0.1 & 0 \end{pmatrix}$

Table: Approximate magnitudes of the neutrino mass matrix elements with $\lambda \sim 0.01$.

Symmetry Realization

- To obtain the desired texture structures for class B, we use the framework of type-I+II seesaw mechanism.
- The type-I seesaw contribution to the effective neutrino mass matrix is given by

$$M_\nu^I \approx -M_D M_R^{-1} M_D^T$$

where M_D and M_R are the Dirac and the right-handed neutrino mass matrices, respectively.
- For the type-II seesaw contribution to the effective neutrino mass matrix, we need a scalar $SU(2)_L$ triplet Higgs ($\triangle$).
- The effective neutrino mass matrix containing both type-I+II seesaw contributions is given by

$$M_\nu \approx M_\nu^{II} + M_\nu^I = M_L - M_D M_R^{-1} M_D^T$$

where M_L denotes the type-II seesaw contribution.

Symmetry Realization

- For the symmetry realization, we use the discrete cyclic group Z_3, which has order 3 with elements: 1, ω, ω^2 where $\omega = e^{i2\pi/3}$.
- For class B_1, we assume the following transformation properties of the leptonic fields under the action of Z_3 symmetry:

$$\begin{aligned} D_{eL} &\to \omega D_{eL}, & e_R &\to \omega e_R, & \nu_{eR} &\to \omega \nu_{eR}, \\ D_{\mu L} &\to \omega^2 D_{\mu_L}, & \mu_R &\to \omega^2 \mu_R, & \nu_{\mu R} &\to \omega^2 \nu_{\mu R}, \\ D_{\tau L} &\to D_{\tau L}, & \tau_R &\to \tau_R, & \nu_{\tau R} &\to \nu_{\tau R}, \end{aligned}$$

where $D_{lL} \equiv \begin{pmatrix} \nu_{lL} \\ l_L \end{pmatrix}$ with $l = e, \mu, \tau$ denotes $SU(2)_L$ doublets and l_R, ν_{lR} denote the right-handed $SU(2)_L$ singlet charged lepton and neutrino fields, respectively.

Symmetry Realization

According to these transformations of the leptonic fields, the bilinears $\overline{D}_{lL} l_R$, $\overline{D}_{lL}\nu_{lR}$ and $\nu_{lR}^T C^{-1}\nu_{lR}$ relevant for M_l, M_D and M_R respectively, transform as

$$\overline{D}_{lL} l_R \sim \overline{D}_{lL}\nu_{lR} \sim \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{pmatrix}, \quad \nu_{lR}^T C^{-1}\nu_{lR} \sim \begin{pmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{pmatrix}.$$

The SM Higgs doublet is assumed to remain invariant under Z_3 leading to diagonal M_l, M_D. M_R has the following form:

$$M_R = \begin{pmatrix} 0 & C & 0 \\ C & 0 & 0 \\ 0 & 0 & D \end{pmatrix}.$$

This leads to the following type-I seesaw contribution to the effective neutrino mass matrix:

$$M_\nu^I = \begin{pmatrix} 0 & c & 0 \\ c & 0 & 0 \\ 0 & 0 & d \end{pmatrix}.$$

Symmetry Realization

- We add a Higgs triplet $\triangle$ which transforms as $\triangle \to \omega\triangle$ under Z_3, leading to the following type-II seesaw contribution to the effective neutrino mass matrix:

$$D_{lL}^T C^{-1} D_{lL} \sim \begin{pmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{pmatrix}, \qquad M_\nu^{II} = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}.$$

- The effective neutrino mass matrix after the complete type-I+II seesaw contributions has the form of class B_1 viz.

$$M_\nu \equiv M_\nu^I + M_\nu^{II} = \begin{pmatrix} a & c & 0 \\ c & 0 & b \\ 0 & b & d \end{pmatrix}.$$

Class	$D_{eL},\ D_{\mu L},\ D_{\tau L}$	$e_R,\ \mu_R,\ \tau_R$	$\nu_{eR},\ \nu_{\mu R},\ \nu_{\tau R}$	ϕ	$\triangle$	χ
B_1	$\omega,\ \omega^2,\ 1$	$\omega,\ \omega^2,\ 1$	$\omega,\ \omega^2,\ 1$	1	ω	-
B_2	$\omega^2,\ 1,\ \omega$	$\omega^2,\ 1,\ \omega$	$\omega^2,\ 1,\ \omega$	1	ω^2	-
B_3	$1,\ \omega^2,\ \omega$	$1,\ \omega^2,\ \omega$	$\omega,\ 1,\ 1$	1	1	ω^2
B_4	$1,\ \omega,\ \omega^2$	$1,\ \omega,\ \omega^2$	$\omega,\ 1,\ 1$	1	1	ω^2

Table: The transformation properties of lepton and scalar fields under Z_3 for classes B_1, B_2, B_3 and B_4.

Symmetry Realization

	M_D	M_R	M_ν^I	M_ν^{II}	M_ν
B_1	$\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$	$\begin{pmatrix} 0 & C & 0 \\ C & 0 & 0 \\ 0 & 0 & D \end{pmatrix}$	$\begin{pmatrix} 0 & c & 0 \\ c & 0 & 0 \\ 0 & 0 & d \end{pmatrix}$	$\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$	$\begin{pmatrix} a & c & 0 \\ c & 0 & b \\ 0 & b & d \end{pmatrix}$
B_2	$\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & C \\ 0 & D & 0 \\ C & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & c \\ 0 & d & 0 \\ c & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$	$\begin{pmatrix} a & 0 & c \\ 0 & d & b \\ c & b & 0 \end{pmatrix}$
B_3	$\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ z & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & A & B \\ A & C & D \\ B & D & E \end{pmatrix}$	$\begin{pmatrix} c & 0 & d \\ 0 & 0 & 0 \\ d & 0 & e \end{pmatrix}$	$\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$	$\begin{pmatrix} a+c & 0 & d \\ 0 & 0 & b \\ d & b & e \end{pmatrix}$
B_4	$\begin{pmatrix} 0 & x & y \\ z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & A & B \\ A & C & D \\ B & D & E \end{pmatrix}$	$\begin{pmatrix} c & d & 0 \\ d & e & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$	$\begin{pmatrix} a+c & d & 0 \\ d & e & b \\ 0 & b & 0 \end{pmatrix}$

Table: Structures of M_D, M_R, type-I and type-II seesaw contributions to the effective neutrino mass matrix.

Stability of Texture Zeros

- All these texture zeros are realized at the seesaw scale which poses the question whether the texture zeros realized in this work survive when the RG evolution of M_ν from the seesaw to the electroweak scale is taken into account.
- At one loop, the RG running of the neutrino mass matrix below the seesaw scale is described by the RG equation

$$16\pi^2 \frac{\mathrm{d}\kappa}{\mathrm{d}t} = C(Y_l Y_l^\dagger)\kappa + C\kappa(Y_l Y_l^\dagger) + \xi\kappa$$

  where κ denotes the effective dimension five neutrino mass operator, Y_l is the Yukawa coupling matrix for the charged leptons,
- From above equation one can see that in the flavor basis where the charged lepton Yukawa coupling matrix is diagonal, the radiative corrections to each element of the effective neutrino mass matrix are multiplicative, so that a zero entry remains zero.

Stability of Texture Zeros

The situation changes at energies larger than the mass scale of the lightest right-handed neutrino. Above this threshold, the neutrino Yukawa couplings Y_ν also contribute to the RG equations.

- Above the highest seesaw scale, the effective neutrino mass matrix is defined as

$$M_\nu = -\frac{v^2}{2} Y_\nu M_R^{-1} Y_\nu^T$$

  Between the mass thresholds, the singlet neutrinos are successively integrated out which leads to modifications in running.

- In the full type-I+II seesaw scenario, the running of M_ν above and between the seesaw scales is given by the running of following three different contributions to M_ν:

$$M_\nu^{(1)} = -\frac{v^2}{4}\kappa, \quad M_\nu^{(2)} = -\frac{v^2}{2} Y_\nu M_R^{-1} Y_\nu^T, \quad M_\nu^{(3)} = \frac{v^2}{2} \Lambda M_\triangle^{-2} Y_\triangle$$

  where $\Lambda, M_\triangle >> v$.

Stability of Texture Zeros

- The one loop β-functions for the effective neutrino mass matrix in various effective theories can be summarized as

$$16\pi^2 \frac{\mathrm{d}M_\nu^{(i)}}{\mathrm{d}t} = [C_l Y_l Y_l^\dagger + C_\nu Y_\nu Y_\nu^\dagger + C_\triangle Y_\triangle Y_\triangle^\dagger]^T M_\nu^{(i)} + M_\nu^{(i)}[C_l Y_l Y_l^\dagger + C_\nu Y_\nu Y_\nu^\dagger + C_\triangle Y_\triangle Y_\triangle^\dagger] + \xi M_\nu^{(i)}$$

  where $M_\nu^{(i)}$ denotes any of the three contributions to the effective neutrino mass matrix.
- For all the Yukawa coupling matrices realized in this work, the hermitian products $Y_k Y_k^\dagger$ $(k = l, \nu, \triangle)$ which are relevant for the RG evolution of M_ν come out to be diagonal.
- Due to diagonal $Y_k Y_k^\dagger$ the RG corrections are again multiplicative on the effective neutrino mass matrix elements leaving zero elements intact.
- Thus, the correlations induced between neutrino masses and mixing parameters by texture zeros remain unchanged due to the stability of texture zeros.

Summary

- We have studied the implications of large $|M_{ee}|$ for a class of two texture zero neutrino mass matrices.
- We found that Classes B_1, B_2, B_3 and B_4 all predict near maximal atmospheric neutrino mixing angle when supplemented with the assumption of large $|M_{ee}|$.
- The near maximality of θ_{23} is independent of the values of θ_{12} and θ_{13}.
- One can obtain such texture structures in the context of type-I+II seesaw using the discrete Abelian group Z_3.
- The texture zeros realised in this work remain stable under RG running of the effective neutrino mass matrix from the seesaw scale to the electroweak scale, at one loop level.
- There are a number of forthcoming and presently ongoing experiments searching for neutrinoless double beta decay. These experiments are capable of confirming or ruling out large $|M_{ee}|$.

Thank You For Your Attention!

Extra Slides

Type-I Seesaw

- The Dirac neutrino mass matrix (M_D) along with the right handed Majorana mass matrix (M_R) forms a Majorana mass term for neutrino fields:

$$\mathcal{L}_{M_D+M_R} = \frac{1}{2} N_L^T C^{-1} M N_L + \text{ H. c.}$$

with $N_L = (\nu_L, (\nu_R)^c)^T = (\nu_L, \nu_L^c)^T$.

- The 6×6 mass matrix M has the form

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}.$$

- For $M_R \gg M_D$, Block diagonalization of M yields:

$$M_\nu \approx -M_D M_R^{-1} M_D^T.$$

the smallness of M_ν is a direct consequence of the large mass scale of M_R.

Type-II Seesaw

- Instead of extending the SM by adding right-handed neutrinos, one can add an $SU(2)_L$ triplet Higgs $(\delta_1, \delta_2, \delta_3)^T$ to the SM to form renormalizable mass term for neutrinos.
- The triplet in 2×2 matrix notation is

$$\Delta = \sum_{i=1}^{3} \delta_i \tau_i = \begin{pmatrix} H^+ & \sqrt{2}H^{++} \\ \sqrt{2}H^o & -H^+ \end{pmatrix}$$

  τ_i are the Pauli matrices.
- The relevant terms of the Lagrangian are

$$\mathcal{L}_{M_L} = \cdots + \left(\sum_{j,k=e,\mu,\tau} \frac{y^{\Delta}_{jk}}{2} D^T_{j_L} C^{-1} i\tau_2 \Delta D_{k_L} + \text{ H. c.} \right) -$$

$$M^2_{\Delta} \mathrm{Tr}[\Delta^\dagger \Delta] \quad - \left(\mu_\Delta \phi^\dagger \Delta \tilde{\phi} + \text{ H. c.} \right) - \cdots$$

Type-II Seesaw

- Neutrino masses are generated when the neutral component of the Higgs triplet develops a small VEV

$$M_L = y^{\Delta}_{jk} \frac{v_\Delta}{\sqrt{2}}, \quad \langle \Delta \rangle_o = \begin{pmatrix} 0 & 0 \\ v_\Delta & 0 \end{pmatrix}.$$

- Generation of small neutrino masses requires a small triplet VEV:

$$|v_\Delta| \simeq \frac{|\mu_\Delta| v^2}{M_\Delta^2}.$$

  which in turn requires a new heavy scale (M_Δ) in the scalar sector.
- When both, right-handed neutrinos and $SU(2)_L$ triplet Higgs are present, we get type-(I+II) seesaw with the seesaw formula

$$M_\nu = M_L - M_D M_R^{-1} M_D^T.$$

- In the case of two texture zeros, there exists a permutation symmetry between different patterns. This corresponds to the permutation of the 2-3 rows and 2-3 columns of M_ν. The corresponding permutation matrix is given by

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

- This leads to the following relations between the parameters of the classes related by the permutation symmetry:

$$\theta_{12}^X = \theta_{12}^Y,\ \theta_{13}^X = \theta_{13}^Y,\ \theta_{23}^X = \frac{\pi}{2} - \theta_{23}^Y,\ \delta^X = \delta^Y - \pi$$

  where X and Y denote classes related by the permutation symmetry.

- The classes related by the 2-3 permutation symmetry are

$$A_1 \leftrightarrow A_2,\ B_1 \leftrightarrow B_2,\ B_3 \leftrightarrow B_4$$

  whereas the class C transforms unto itself.