Two dimensional hydrodynamics with gauge and gravitational anomalies

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References

- **R. Banerjee,** "Exact results in two dimensional chiral hydrodynamics with gravitational anomalies," **arXiv:1303.5593 [gr-qc].Eur. Phys. J. C 74, 2824 (2014);**
- **R. Banerjee, S. Dey, B. R. Majhi, A. K. Mitra;** " Two dimensional hydrodynamics with gauge and gravitational anomalies," **arXiv:1307.1313 [gr-qc]. Phys Rev.D 89, 104013(2014);**
- **R. Banerjee, S. Dey.** " Constitutive relations and response parameters in two dimensional hydrodynamics with gauge and gravitational anomalies," **arXiv:1403.7357[gr-qc], Phys. Lett. B733 (2014) 198;**

Modern applications of the chiral anomaly

- **1. Quantum wires**
- **2. Quantum Hall effect**
- **3. Hawking effect**
- **4. Chiral magnetic effect**
- **5. Chiral vortical effect**
- **6. Anomalous hydrodynamics**

Relativistic Fluid Dynamics

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- Large velocity (comparable to light) of macroscopic flow.
- Microscopic motion of fluid particles is large.

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- **Equation of Motion:**

$$\partial_\mu T^\mu_\nu = 0$$

- Conservation of Energy-momentum tensor.
- For a charged fluid this is supplemented with

$$\partial_\mu J^\mu = 0$$

Constitutive Relations:

- Additional relations expressing E.M tensor/Charge in terms of the basic fluid variables like velocity, temperature and chemical potential.

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- **Ideal Fluid Relations:**

$$T_{\mu\nu} = (\varepsilon + \mathcal{P})\, u_{\mu\nu} + \mathcal{P}\eta_{\mu\nu} \ ,$$

$$J_\mu = n u_\mu$$

- $\varepsilon \rightarrow$ energy density, $\mathcal{P} \rightarrow$ pressure, $n \rightarrow$ charge density, $\eta_{\mu\nu} \rightarrow$ metric, $u_\mu \rightarrow$ fluid velocity normalised as $u^\mu u_\mu = -1$.
- Extra terms have to be included in the non ideal case to include effects of dissipation (like viscosity).

Two Approaches

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  Constitutive relations are derived to ensure positivity of entropy and hence compatibility with a local version of the second law of thermodynamics. Also, it satisfies the appropriate equations of motion.

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- **Derivative expansion approach:**
  Effective action is expressed as a series in powers of derivatives acting on fluid variables (like velocity). This is the large wavelength approximation.
  Likewise, the constitutive relations are also expressed as a power series.

  Results from these approaches agree although a general proof of this statement is missing.

Hydrodynamics in presence of gauge/gravity

Turn on a gauge field (A_μ) and gravity (metric $g_{\mu\nu}$).

- **Changes**

  Replace ordinary derivative by covariant derivative in the conservation laws

$$D_\mu T^\mu_\nu = 0, \qquad D_\mu J^\mu = 0;$$

  Modify constitutive relations:

  Depend on gauge and/or diffeomorphism invariant combinations of the fields ($A_\mu, g_{\mu\nu}$).

  What happens if anomalies are present?

$$D_\mu T^\mu_\nu \neq 0, \qquad D_\mu J^\mu \neq 0;$$

A hydrodynamic (derivative) expansion is usually adopted

Review on anomalies

- **Standard definition**

  Anomaly is the breakdown of a classical symmetry upon quantization.

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- **Example:**

  QED

$$\partial_\mu J^\mu = 0, \qquad \partial_\mu J^{\mu 5} = 0;$$

Both vector/axial vector currents are conserved. Results follow on using the classical equation of motion(Noether's theorem).

More refined calculation yields,

$$\partial_\mu J^\mu = 0, \qquad \partial_\mu J^{\mu 5} = \frac{1}{16\pi^2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta};$$

Infinities and Anomalies

Anomaly is the breakdown of formal manipulations(ignoring infinities) leading to a modified conservation law.

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- **Same example (QED)**:

$$\partial_\mu J^\mu(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\psi\right) = \bar{\psi}\overrightarrow{\not{\partial}}\psi + \bar{\psi}\overleftarrow{\not{\partial}}\psi$$

- **Classical equations of motion**

$$\overrightarrow{\not{\partial}}\psi = m\psi, \quad \bar{\psi}\overleftarrow{\not{\partial}} = -m\bar{\psi}$$

$$\partial_\mu J^\mu(x) \quad = \quad m\bar{\psi}(x)\psi(x) - m\bar{\psi}(x)\psi(x)$$

$$=0$$

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Allowed only if $\bar\psi(x)\psi(x)$ is not infinity!

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$$\langle T\bar\psi(x)\psi(y)\rangle \quad = \quad \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip.(x-y)}}{\not p - m}$$

- for x=y $\int \frac{d^4p}{(2\pi)^4}\frac{1}{\not p - m} \rightarrow$ divergent at $p \rightarrow \infty$

Chiral anomaly

Anomaly in the chiral current

$$\partial_\mu \left[\bar{\psi} \gamma^\mu \left(\frac{1 \pm \gamma^5}{2} \right) \psi \right] \quad = \quad A$$

No regularisation exists for which $A = 0$

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- **Covariant and consistent anomaly**

- Current transforming covariantly under a gauge transformation is called covariant current.
  Anomaly of a covariant current also transforms covariantly $\rightarrow$ Covariant anomaly.

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- **Covariant and consistent anomaly**

- Current transforming covariantly under a gauge transformation is called covariant current.
  Anomaly of a covariant current also transforms covariantly $\rightarrow$ Covariant anomaly.
- Current defined from the variation of an effective action is called consistent current.

  Anomaly of consistent current is consistent anomaly (satisfies W-Z consistency condition).

  Covariant and consistent expressions are complementary, related by local polynomials.

Example from 2 dimensions(chiral gauge anomaly)

Covariant anomaly

$$\partial_\mu J^\mu = \frac{1}{4\pi} \epsilon_{\mu\nu} F^{\mu\nu}$$

- **Effective action**

$$W[A] = \int_0^1 dg \int d^2x \; A_\mu(x) J^{\mu(g)}(x)$$

Formal definition is made sensible by specifying a regularisation for $J^{\mu(g)}(x)$.

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- **Choose a gauge invariant regularisation**

$$W[A - \partial\alpha] = \int_0^1 dg \int d^2x \left(A_\mu - \partial_\mu \alpha\right) J^{\mu(g)}$$

$$\int d^2x \left(\partial_\mu \frac{\delta W}{\delta A_\mu}\right)\alpha = \int_0^1 dg \int d^2x \; \alpha(x) \; \partial_\mu J^{\mu(g)}$$

$$\partial_\mu \frac{\delta W}{\delta A_\mu} = \int_0^1 dg \left(\frac{g}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}\right) = \frac{1}{8\pi}\epsilon_{\mu\nu}F^{\mu\nu}$$

This is the consistent anomaly.

Example continued

Covariant: $\frac{1}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}$; Consistent: $\frac{1}{8\pi}\epsilon_{\mu\nu}F^{\mu\nu}$

- **For any d=2n dimensions**

  Consistent anomaly = $\frac{1}{n+1}$ Covariant anomaly

  Follows from homogeneous nature of anomaly

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- **Relation between covariant and consistent currents**

$$J_\mu^{Cov} = J_\mu^{Const} + \frac{1}{4\pi}\epsilon_{\mu\nu}A^\nu$$

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- **Relation between covariant and consistent currents**

$$J_\mu^{Cov} = J_\mu^{Const} + \frac{1}{4\pi}\epsilon_{\mu\nu}A^\nu$$

- Extra piece (local polynomial) does not contribute to the effective action.

$$\int_0^1 dg \int d^2x \; A_\mu \left(\frac{1}{4\pi}\right) \epsilon^{\mu\nu} A_\nu = 0$$

W is therefore unaffected by the regularisation prescription.

Gravitational anomaly(2 dimensions)

- Occurs in (4n-2) dimension. (2,6,10,..)
- For a usual (non-chiral) theory, one can trade between the conformal (trace) and general coordinate (diffeomorphism) symmetries.

$$T^{\mu}_{\mu} = 0,\ \nabla_{\mu} T^{\mu}_{\nu} \neq 0 \ \ \text{OR} \ \ T^{\mu}_{\mu} \neq 0,\ \nabla_{\mu} T^{\mu}_{\nu} = 0$$

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- As in the gauge theory, here also there are covariant and consistent expressions for the diffeomorphism anomaly.

Anomalous Ward identities

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Follows from purely algebraic arguments. J^μ, $T^{\mu\nu}$ are covariant current/stress tensor, R is the Ricci scalar, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is field strength

General set up(1+1 dimensional static space-time)

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- **Antisymmetric tensor**:

$$\bar{\epsilon}_{\mu\nu} = \sqrt{-g}\epsilon_{\mu\nu} = \frac{e^{2\sigma}}{2}\epsilon_{\mu\nu}$$

$$\epsilon_{uv} = -\epsilon_{vu} = 1$$

Passage to hydrodynamics

- Introduce the velocity u^μ of the time independent equilibrium fluid fields, satisfying $u^\mu u_\mu = \ -1$ (comoving frame)

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- **Chiral vector**

$$u^c_\mu = u_\mu - \tilde{u}_\mu \; = -\bar{\epsilon}_{\mu\nu} u^{\nu c}$$

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- **Chemical potential**

$$\mu = A_t(r)/\sqrt{-g_{00}} = A_t(r)e^{-\sigma}$$

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$$T = T_0 e^{-\sigma}$$

where T_0 is the equilibrium temperature

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- **Ricci scalar**

$$R = \frac{1}{g_{11}^2}(g_{11}^{'}\sigma' - 2g_{11}\sigma'^2 - 2g_{11}\sigma^{''}) \; = -2u^\mu \nabla^\nu \nabla_\mu u_\nu$$

Constitutive relations

- **Energy-momentum Tensor:**

$$
\begin{aligned}
T_{\mu\nu} = &\left[C_1 T^2 - C_w \left(u^\alpha \nabla^\beta \nabla_\beta u_\alpha\right) + \mu^2 \left(\tfrac{1}{2\pi} - C_s\right)\right] g_{\mu\nu} \\
&+ \left[2C_w \left(u^\alpha \nabla^\beta - u^\beta \nabla^\alpha\right) \nabla_\alpha u_\beta + 2C_1 T^2 + 2\mu^2 \left(\tfrac{1}{2\pi} - C_s\right)\right] u_\mu u_\nu \\
&- \left[2C_g \left(u^\alpha \nabla^\beta - u^\beta \nabla^\alpha\right) \nabla_\alpha u_\beta + C_2 T^2 + C_s \mu^2\right] \left(u_\mu \tilde{u}_\nu + \tilde{u}_\mu u_\nu\right) \\
&+ \left\{ \left(\tfrac{C}{\pi} - 2(C+P)C_s\right) \tfrac{T}{T_0} \mu + \left(\frac{C^2+P^2}{2\pi} - C_s (C+P)^2\right) \tfrac{T^2}{{T_0}^2} \right\} \left(2u_\mu u_\nu + g_{\mu\nu}\right) \\
&+ \left\{ \left(\tfrac{P}{\pi} - 2(C+P)C_s\right) \tfrac{T}{T_0} \mu + \left(\tfrac{CP}{\pi} - C_s (C+P)^2\right) \tfrac{T^2}{{T_0}^2} \right\} \left(u_\mu \tilde{u}_\nu + \tilde{u}_\mu u_\nu\right)
\end{aligned}
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& + \left[2C_w \left(u^\alpha \nabla^\beta - u^\beta \nabla^\alpha\right) \nabla_\alpha u_\beta + 2C_1 T^2 + 2\mu^2 \left(\tfrac{1}{2\pi} - C_s\right)\right] u_\mu u_\nu \\
& - \left[2C_g \left(u^\alpha \nabla^\beta - u^\beta \nabla^\alpha\right) \nabla_\alpha u_\beta + C_2 T^2 + C_s \mu^2\right] \left(u_\mu \tilde{u}_\nu + \tilde{u}_\mu u_\nu\right) \\
& + \left\{\left(\tfrac{C}{\pi} - 2(C+P)C_s\right) \tfrac{T}{T_0} \mu + \left(\tfrac{C^2+P^2}{2\pi} - C_s (C+P)^2\right) \tfrac{T^2}{T_0{}^2}\right\} \left(2u_\mu u_\nu + g_{\mu\nu}\right) \\
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\end{aligned}
$$

- **Gauge current**

$$
\begin{aligned}
J_\mu = -2C_s \mu \left(u_\mu + \tilde{u}_\mu\right) + \frac{\mu}{\pi} u_\mu + \left(\frac{C}{\pi} - 2(C+P)C_s\right) \frac{T}{T_0} u_\mu \\
+ \left(\frac{P}{\pi} - 2(C+P)C_s\right) \frac{T}{T_0} \tilde{u}_\mu,
\end{aligned}
$$

Here, C_1, C_2,P and C are constants.

Derivative expansion approach

- **Covariant stress tensor**

$$T^{\mu\nu} \;=\; \varepsilon u^\mu u^\nu + \mathcal{P}\tilde{u}^\mu \tilde{u}^\nu + \theta\,(\tilde{u}^\mu u^\nu + u^\mu \tilde{u}^\nu)$$

- General form of a symmetric second rank tensor constructed from u_μ and $\tilde{u}_\mu$.

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$$\begin{aligned}
\varepsilon &= C_1 T^2 + C_w\left(u^\nu \nabla^\mu \nabla_\mu u_\nu\right) + 2C_w\left(u^\mu \nabla^\nu - u^\nu \nabla^\mu\right)\nabla_\mu u_\nu \\
\mathcal{P} &= C_1 T^2 - C_w\left(u^\nu \nabla^\mu \nabla_\mu u_\nu\right) \\
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\end{aligned}$$

- C_1 and C_2 are undetermined parameters expressed in terms of the normalisation factors (C_w, C_g) of the trace/diffeomorphism anomalies. Non-trivial relations,

$$C_1 = 4\pi^2 C_w, \quad C_2 = 8\pi^2 C_g$$

Israel-Hartle-Hawking condition

- Defined by taking $T_{\mu\nu}/J_{\mu}$ in Kruskal coordinates, corresponding to both outgoing and ingoing modes, as regular, near the horizon.

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- Implies $T_{uu},\ T_{vv},\ J_u,\ J_v \to 0$ near the horizon
- Horizon $(r = r_0)$ defined as $e^{2\sigma}|_{r_0} = \frac{1}{g_{11}}|_{r_0} = 0$

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- Fixes all the undetermined constants:

  C and P fixed from $J_u,\ J_v \to 0$

  For $J_u \to 0, \quad P - C = \mu e^{\sigma}|_{r_0} = \ 0$

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  P=C=0;

Israel-Hartle-Hawking condition

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  $\mathbf{P=C=0}$;
- C_1 and C_2 fixed from the condition on stress tensor.

$$\mathbf{C_1 = 4\pi^2 C_w, \quad C_2 = 8\pi^2 C_g,}$$

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- **Comparison yields**

$$\frac{\partial P}{\partial \mu} = T^2\frac{\partial p_0}{\partial \mu} = \left(-2C_s + \frac{1}{\pi}\right)\mu; \quad a_2 = a'_2 = 0$$

Final Expressions

- $$p_0 = \left(\frac{1}{\pi} - C_s\right)\frac{\mu^2}{T^2} + Q(int.const)$$
- In the absence of gauge field
$$p_0 = C_1 = 4\pi^2 C_w$$
- General solution:
$p_0 = 4\pi^2 C_w + \left(\frac{1}{\pi} - C_s\right)\frac{\mu^2}{T^2}$

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- **Consistency check**

  The constitutive relation for $T_{\mu\nu}$ agrees with the form obtained by the derivative expansion provided the above identifications are used.

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Thank You