Is the size of θ_{13} related to the smallness of the solar mass splitting?

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Biswajoy Brahmachari and A.R., Phys. Rev D 86, 051302(R) (2012)

Soumita Pramanick and A.R., Phys. Rev D 88, 093009 (2013) and work in progress

Plan

Background: The mixing angles and mass splittings

- The PMNS matrix and a few commonly considered mixing patterns
- $\theta_{13} \neq 0$: small compared to other mixing angles
- The mass splittings: solar splitting much smaller than atmospheric

Are θ_{13} and the solar splitting both generated by a perturbation?

- Perturbation theory for θ_{13} and solar splitting
- Ensure θ_{12} within 1σ of best-fit
- CP-violation prediction
- A Mass Model

Conclusions

The PMNS mixing matrix

Three flavours: ν_e, ν_μ, ν_τ

2 independent Δm^2, 3 mixing angles, 1 phase + 2 Majorana phases

$$U^T M_\nu U = \mathrm{diag}(m_1, m_2, m_3)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

CP-violation measure: $J = \mathrm{Im}[U_{e1}U_{\mu 2}U^*_{e2}U^*_{\mu 1}]$

Oscillation probability

$$P_{\nu_\alpha \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i}U_{\beta i}U^*_{\alpha j}U^*_{\beta j} \sin^2\left(\frac{\pi L}{\lambda_{ij}}\right)$$

Some popular mixing patterns

General parametrisation: $\theta_{13} = 0, \ \theta_{23} = \pi/4$

$$U^0 = \begin{pmatrix} \cos\theta^0_{12} & \sin\theta^0_{12} & 0 \\ -\sin\theta^0_{12}/\sqrt{2} & \cos\theta^0_{12}/\sqrt{2} & \sqrt{\frac{1}{2}} \\ \sin\theta^0_{12}/\sqrt{2} & -\cos\theta^0_{12}/\sqrt{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

Popular choices

$$U^0 = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \\ \frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}, \begin{pmatrix} \sqrt{\frac{\phi}{\sqrt{5}}} & \sqrt{\frac{1}{\sqrt{5}\phi}} & 0 \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & \frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & \sqrt{\frac{1}{2}} \\ \frac{1}{\sqrt{2}}\sqrt{\frac{1}{\sqrt{5}\phi}} & -\frac{1}{\sqrt{2}}\sqrt{\frac{\phi}{\sqrt{5}}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

Tribimaximal mixing | Bimaximal mixing | Golden Ratio $\phi = (1+\sqrt{5})/2$

$\sin\theta^0_{12}$: 0.577 (TBM), 0.707 (BM), 0.526 (GR), $\sin\theta_{12}$: 0.539 $\leftrightarrow$ 0.561 (data at 1 σ)

$\sin\theta_{13} = 0$ in all three scenarios

$\theta_{13} \neq 0$

However, recent (2012) results indicate that $\theta_{13} \neq 0$

$$\begin{aligned} \sin^2 2\theta_{13} &= 0.090^{+0.008}_{-0.009} \text{ (Daya Bay : 217 days, Rate and Spectrum)} \\ &= 0.100 \pm 0.010 \text{ (stat)} \pm 0.015 \text{ (syst) (RENO : 403 live days, Rate only)} \end{aligned}$$

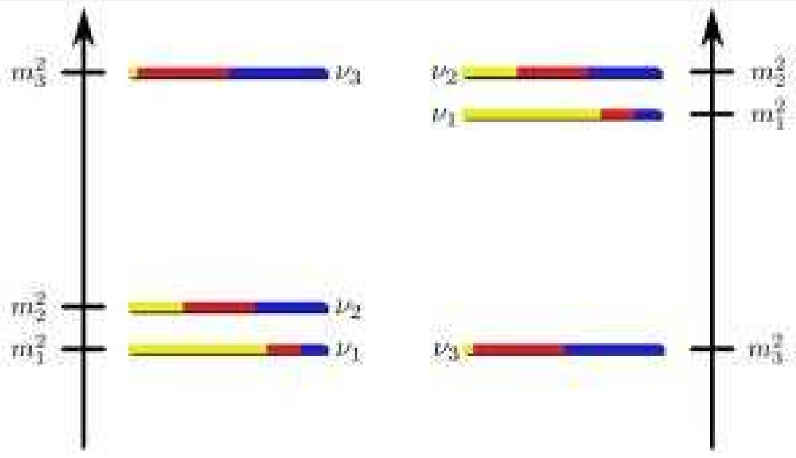
The simplest mixing pictures need to be supplemented.

- Is θ_{13} large? Yes, it is close to the upper limit from prior global fits!
- Is θ_{13} small? Yes, compared to the other mixing angles which are large. Small in an absolute sense.

Mass splitting, mass ordering

Sign of Δm^2_{31} not determined.
Solar neutrinos (matter effect) $\Rightarrow \Delta m^2_{21} > 0$.

Two mass orderings possible:



Solar splitting two orders smaller than atmospheric

Normal $m^- > 0$

Inverted $m^- < 0$

Here, $m^- = m_3 - m_1$
Sign of m^- is one of the most important unsettled issues in neutrino physics.

Best-fit values

Global best-fit values:

$$
\begin{aligned}
\Delta m^2_{21} &= (7.50^{+0.18}_{-0.19}) \times 10^{-5}\,\mathrm{eV}^2, \;\; \theta_{12} = (33.36^{+0.81}_{-0.78})^\circ, \\
|\Delta m^2_{31}| &= (2.473^{+0.070}_{-0.067}) \times 10^{-3}\,\mathrm{eV}^2, \;\; \theta_{23} = (40.0^{+2.1}_{-1.5} \oplus 50.4 \pm 0.13)^\circ \\
\theta_{13} &= (8.66^{+0.44}_{-0.46})^\circ, \;\; \delta = (300^{+66}_{-138})^\circ \;\; .
\end{aligned}
$$

Octant of θ_{23} undetermined.

From: Gonzalez-Garcia, Maltoni, Salvado, and Schwetz, JHEP 12 (2012) 123

Is it possible that the small θ_{13} *and* the small solar mass splitting are both due to a perturbation on a more symmetric structure?

Perturbation Theory

Zero order form

Flavour basis: Charged lepton mass matrix is diagonal

Unperturbed:

Guided by Tribimaximal/Bimaximal/Golden Ratio mixings

Take $\theta_{13} = 0, \quad \theta_{23} = \pi/4$

Further, take solar splitting as absent:

$$(M^0)^{mass} = diag(m_1^{(0)}, m_1^{(0)}, m_3^{(0)}), \quad \text{mass basis.}$$

$m_i^{(0)}$ chosen real and positive

Flavour basis:

$$(M^0)^{flavour} = U^0 \begin{pmatrix} m_1^{(0)} & & \\ & m_1^{(0)} & \\ & & m_3^{(0)} \end{pmatrix} U^{0T}$$

The columns of U^0 are the neutrino flavour eigenstates before perturbation.

The perturbation

Perturbation in the mass basis [$m^{\pm} = m_3^{(0)} \pm m_1^{(0)}$]:

$$M' = m^+ \begin{pmatrix} 0 & \gamma & \xi \\ \gamma & \alpha & \eta \\ \xi & \eta & \beta \end{pmatrix} \qquad (\alpha, \beta \ldots \eta \text{ dimensionless}).$$

Consider two cases: (a) Real perturbation and (b) Complex perturbation.

For real M' Zero order: M^0, Perturbation: M'

For complex M' Zero order: $M^{0\dagger} M^0$, Perturbation: $(M^{0\dagger} M' + M'^{\dagger} M^0)$

Determine $\alpha, \beta, \gamma, \xi, \eta$ by requiring that the mass splitting and mixing angles* agree with the global fits to 1σ.

*Here we keep $\theta_{23} = \pi/4$ fixed.

Solar neutrinos

Degenerate sector of the perturbation:

$$\text{Real } M' : \quad M'_{(2\times 2)} = m^+\alpha \begin{pmatrix} 0 & r \\ r & 1 \end{pmatrix} \quad , \quad r = \gamma/\alpha \quad .$$

$$\text{Complex } M' : \quad (M^{0\dagger} M' + M'^{\dagger} M^0)_{(2\times 2)} = 2m^+ m_1^{(0)} \text{Re}(\alpha) \begin{pmatrix} 0 & r \\ r & 1 \end{pmatrix} \quad , \quad r = \text{Re}(\gamma)/\text{Re}(\alpha) \quad .$$

- Mass splitting:

$$m_{2,1} = m_1^{(0)} + m^+ \frac{\alpha}{2} \left[1 \pm 4r^2\right] \quad \text{Real perturbation}$$

$$m_{2,1}^2 = (m_1^{(0)})^2 + 2m^+ m_1^{(0)} \frac{Re(\alpha)}{2} \left[1 \pm 4r^2\right] \quad \text{Complex perturbation}$$

$$R_{\text{mass}} = |(m_2^2 - m_1^2)/(m_3^2 - m_1^2)| = 2\frac{m_1^{(0)}}{|m^-|} \text{Re}(\alpha)\sqrt{1 + 4r^2} \; .$$

Solar neutrinos (contd.)

- Mixing Angle (nudge θ_{12}):

$$\theta_{12} = \theta_{12}^0 + \zeta \quad \text{where} \quad \zeta = \frac{1}{2}\tan^{-1}(2r) \leftarrow \text{perturbation} \ .$$

Model $\rightarrow$ / 1 σ range $\downarrow$	TBM		BM		GR	
	r_{min}	r_{max}	r_{min}	r_{max}	r_{min}	r_{max}
$r\ (\times 10^2)$	-4.59	-1.95	-23.1	-19.9	1.54	4.18

α and γ fixed from solar ν data

$$|\psi_1\rangle = \cos\zeta \left[\begin{pmatrix} \cos\theta_{12}^0 \\ -\sin\theta_{12}^0/\sqrt{2} \\ \sin\theta_{12}^0/\sqrt{2} \end{pmatrix} - \bar{\xi}\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\right] - \sin\zeta \left[\begin{pmatrix} \sin\theta_{12}^0 \\ \cos\theta_{12}^0/\sqrt{2} \\ -\cos\theta_{12}^0/\sqrt{2} \end{pmatrix} - \bar{\eta}\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\right] ,$$

$$|\psi_2\rangle = \sin\zeta \left[\begin{pmatrix} \cos\theta_{12}^0 \\ -\sin\theta_{12}^0/\sqrt{2} \\ \sin\theta_{12}^0/\sqrt{2} \end{pmatrix} - \bar{\xi}\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\right] + \cos\zeta \left[\begin{pmatrix} \sin\theta_{12}^0 \\ \cos\theta_{12}^0/\sqrt{2} \\ -\cos\theta_{12}^0/\sqrt{2} \end{pmatrix} - \bar{\eta}\begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}\right] ,$$

Perturbation (contd.)

$$\text{Here} \qquad \bar{\xi} = \left(\frac{m^+}{m^-}\right) \xi, \;\; \bar{\eta} = \left(\frac{m^+}{m^-}\right) \eta \text{ for Real } M',$$

$$\bar{\xi} = \left(\frac{m^+}{m^-}\right) \mathrm{Re}(\xi) + i\, \mathrm{Im}(\xi), \;\; \bar{\eta} = \left(\frac{m^+}{m^-}\right) \mathrm{Re}(\eta) + i\, \mathrm{Im}(\eta) \text{ for Complex } M' \,.$$

Similarly

$$|\psi_3\rangle = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + \bar{\xi}^* \begin{pmatrix} \cos\theta_{12}^0 \\ -\sin\theta_{12}^0/\sqrt{2} \\ \sin\theta_{12}^0/\sqrt{2} \end{pmatrix} + \bar{\eta}^* \begin{pmatrix} \sin\theta_{12}^0 \\ \cos\theta_{12}^0/\sqrt{2} \\ -\cos\theta_{12}^0/\sqrt{2} \end{pmatrix} \,.$$

determines θ_{13} and δ

$$\sin\theta_{13}\, e^{-i\delta} = \left[\cos\theta_{12}^0\, \bar{\xi}^* + \sin\theta_{12}^0\, \bar{\eta}^*\right]$$

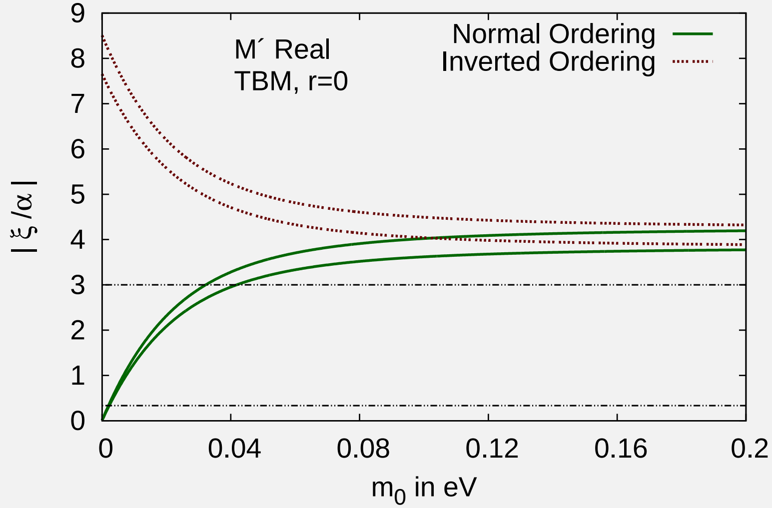
Obtaining $\theta_{13} \neq 0$

$$|\psi_3\rangle = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + \bar{\xi}^* \begin{pmatrix} \cos\theta_{12}^0 \\ -\sin\theta_{12}^0/\sqrt{2} \\ \sin\theta_{12}^0/\sqrt{2} \end{pmatrix} + \bar{\eta}^* \begin{pmatrix} \sin\theta_{12}^0 \\ \cos\theta_{12}^0/\sqrt{2} \\ -\cos\theta_{12}^0/\sqrt{2} \end{pmatrix} .$$

For simplicity keep $\theta_{23} = \pi/4$ unaffected by perturbation $\Rightarrow \left(\bar{\eta}/\bar{\xi}\right)^* = \tan\theta_{12}^0 \Rightarrow$ Real

$$\sin\theta_{13}\, e^{-i\delta} = \left[\cos\theta_{12}^0\, \bar{\xi}^* + \sin\theta_{12}^0\, \bar{\eta}^*\right] = \frac{\bar{\xi}^*}{\cos\theta_{12}^0} ,$$

ξ fixed from θ_{13} and α from $(\Delta m^2)_{solar}$



Real perturbation case

Require elements of perturbation to be roughly of same order: $1/3 \leq |\xi/\alpha| \leq 3$.

This disfavours Inverted ordering.
Normal ordering: $2.3 \leq m_0/(10^{-3}\ \mathrm{eV}) \leq 37$ allowed.

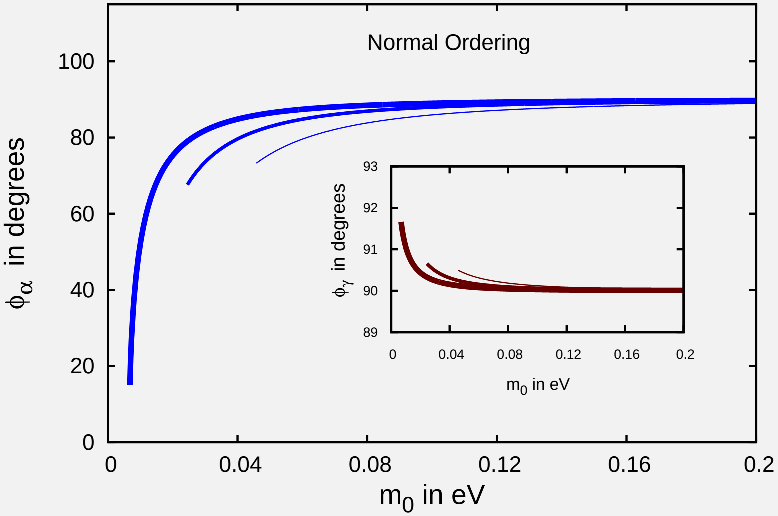
Obtaining $\theta_{13} \neq 0$ and $\delta \neq 0$

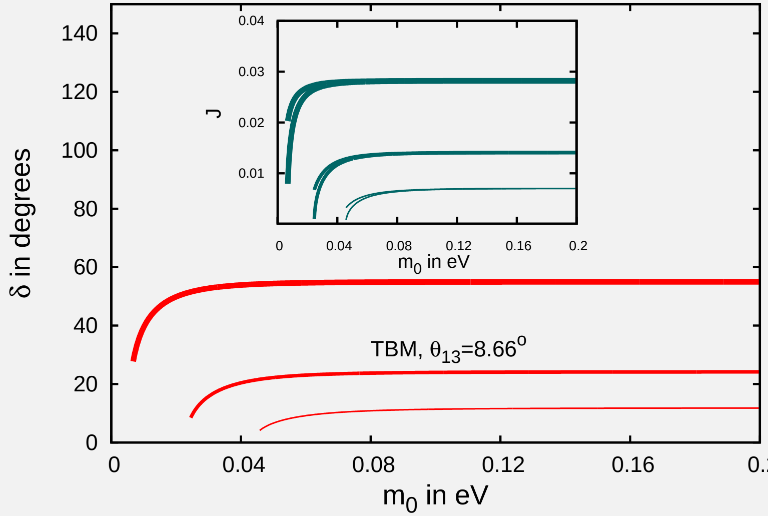
Complex M'

- Substantially increases the number of parameters
- Make a very conservative choice: All matrix elements have the same modulus $\epsilon \leq 1$.
- Only phases of α, γ, and ξ can be different.

$$\sin\theta_{13}\; e^{-i\delta} = \left[\cos\theta_{12}^0\; \bar{\xi}^* + \sin\theta_{12}^0\; \bar{\eta}^*\right] = \frac{\bar{\xi}^*}{\cos\theta_{12}^0}\;,$$

ϕ_α from solar splitting, ϕ_γ from θ_{12}, ϕ_ξ from θ_{13}, and δ is a prediction.





Normal Ordering and TBM

ϵ = 0.1, 0.05, and 0.0.25 in decreasing thickness.

Perturbation magnitude ϵ

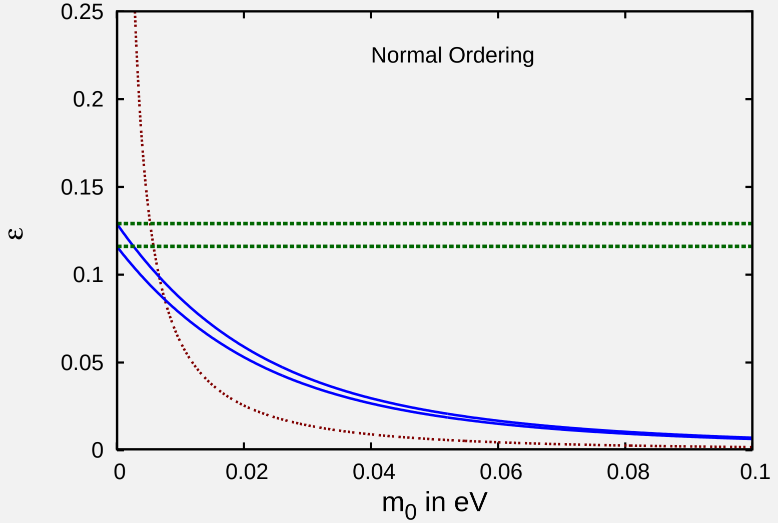
Complex M': All elements have same magnitude ϵ

$$\bar{\xi} = \left(\frac{m^+}{m^-}\right) \mathrm{Re}(\xi) + i\,\mathrm{Im}(\xi) \Rightarrow \left|\frac{m^+}{m^-}\right| \epsilon \geq |\bar{\xi}| \geq \epsilon$$

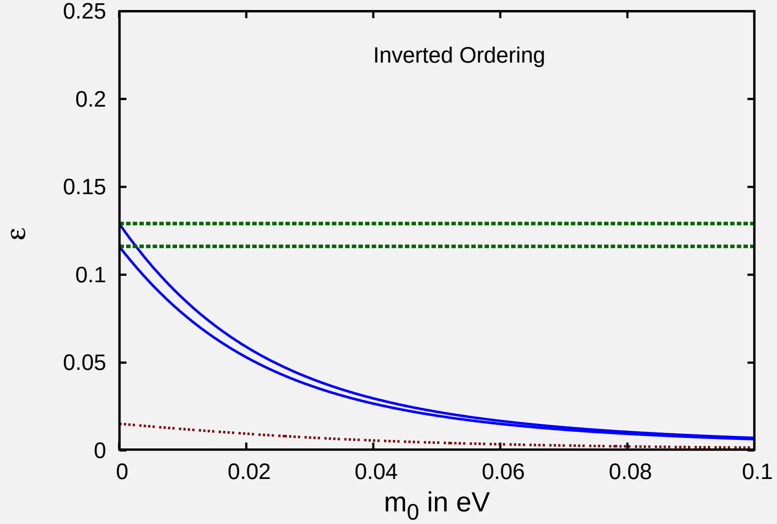
θ_{13} can be reproduced if ϵ is within a fixed range: between the blue solid and green dashed curves.

$(\Delta m^2)_{solar}$ requires ϵ to be larger than the maroon dotted line.

ϵ less than ~ 0.13.



Normal Ordering



Inverted ordering

A Mass Model

Model for the perturbation

- A mass model using Type-I and Type-II see-saw.
- Type-II see-saw with $\mu - \tau$ symmetry produces the unperturbed piece.
- $\theta_{13} = 0, \theta_{23} = \pi/4$, no solar splitting and further $\theta_{12} = 0$.
- Type-I see-saw generates the perturbation.
- Choose Dirac mass term proportional to the identity.
- RH Majorana mass matrix has one neutrino decoupled.
- Result: solar mixing and splitting, θ_{13}, and deviation of θ_{23} from maximality.
- Real perturbation: θ_{12} not within 1σ
- One phase complex perturbation reproduces solar splitting, mixing, and θ_{13}. Also, first octant of θ_{23} for NO and CP-violation.
- Inverted Ordering cannot be accommodated at 1σ.

Type-II see-saw

The unperturbed mass matrix is obtained by a Type-II see-saw: *vev* of a triplet scalar Δ_L.

$$\mathcal{L}_{Type-II} = \frac{1}{2}\sum_{i,j} h_{ij}\,(\nu_L^i)^T C^{-1} \nu_L^j < \Delta_L > + h.c. \quad (i,j = 1,2,3) \quad h_{ij} = h_{ji}, \quad \nu_L \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L$$

a) $\nu_\mu - \nu_\tau$ mixing following $\mu - \tau$ symmetry $\Rightarrow h_{22} = h_{33}$.

b) ν_e unmixed $\Rightarrow h_{12} = h_{13} = 0$. This can be accomplished by a Z_2 symmetry.

$$\mathbb{Z}_2 : \nu_{eL} \to \nu_{eL}; \quad (\nu_{\mu,\tau})_L \to -(\nu_{\mu,\tau})_L; \quad \Delta_L \to \Delta_L$$

$$\text{Mass matrix} \to M_L^{flavour} = \begin{pmatrix} x & 0 & 0 \\ 0 & y & z \\ 0 & z & y \end{pmatrix} \Rightarrow \text{Mixing matrix} \to U^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

Unperturbed: $\theta_{13} = 0$, $\theta_{23} = \pi/4$, $\theta_{12} = 0$

Type-II see-saw (contd.)

In the mass basis

$$M_L^{mass} = \begin{pmatrix} x & 0 & 0 \\ 0 & y-z & 0 \\ 0 & & y+z \end{pmatrix}$$

Thus $m_1^{(0)} \equiv x$, $m_2^{(0)} \equiv (y-z)$, and $m_3^{(0)} \equiv (y+z)$.

No solar splitting: $y - z = x \Rightarrow h_{22} - h_{23} = h_{11}$

Unperturbed: $\theta_{13} = 0, \theta_{23} = \pi/4, \theta_{12} = 0, (\Delta m^2)_{solar} = 0, m_1^0 = m_2^0 = x, m_3^0 = (y+z)$

Perturbation: Type-I see-saw

The Type-I see-saw acts as the perturbation. *vev* of a doublet scalar Φ gives the Dirac mass.

$$\mathcal{L}_{Type-I} = \sum_{i,j} \lambda_{ij}\, \bar{\nu}^i_L N^j_R < \Phi > + \frac{1}{2} \sum_{i,j} H_{ij}\, (N^i_R)^T C^{-1} N^j_R + h.c. \quad , \quad N_R \equiv \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix}_R$$

Take $\lambda_{ij} = \lambda_0 \delta_{ij} \Rightarrow M_D^{flavour} = m_D \mathbb{I}$ and $M_R^{flavour} = m_R \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

$$\text{Perturbation}: \quad M'^{flavour} = M_D^T M_R^{-1} M_D = \frac{m_D^2}{m_R} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

$$\text{In mass basis}: \quad M'^{mass} = \frac{m_D^2}{\sqrt{2} m_R} \begin{pmatrix} 0 & 1 & 1 \\ 1 & \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \\ 1 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} .$$

H_{ij} can be complex. But first we take them real.

Real Perturbation

Recall

$$U^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \\ 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{pmatrix} \; .$$

One gets including the perturbation

$$|\psi_3\rangle = \begin{pmatrix} \sigma \\ \frac{1}{\sqrt{2}}(1 - \frac{\sigma}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}}(1 + \frac{\sigma}{\sqrt{2}}) \end{pmatrix}$$

where $\sigma \equiv \frac{m_D^2}{\sqrt{2} m_R m^-} = s_{13} \cos\delta, \quad \delta = 0 \; (\pi)$ for NO (IO)

$$\tan\theta_{23} = \frac{1 - \frac{\sigma}{\sqrt{2}}}{1 + \frac{\sigma}{\sqrt{2}}} = \tan(45^\circ - \varphi) \quad \text{where} \quad \varphi = \tan^{-1}\left(\frac{s_{13}\cos\delta}{\sqrt{2}}\right) .$$

$(\theta_{23} - \pi/4)$ determined by $s_{13}\cos\delta$

θ_{13} is generated, $\theta_{23} - \pi/4$ related to $s_{13}\cos\delta$, no CP-violation

Real Perturbation (Contd.)

The solar sector: $$M'^{mass}_{2\times 2} = \frac{m_D^2}{\sqrt{2}m_R}\begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix} = \sqrt{2}m^- s_{13}\cos\delta \begin{pmatrix} 0 & 1 \\ 1 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

This implies $\boxed{\Delta m^2_{solar} \equiv m_2^2 - m_1^2 = 3\sqrt{2}\, s_{13}\cos\delta m_1^{(0)} m^-}$ irrespective of m_D, m_R.

The relationship between θ_{13} and the solar splitting restricts allowed m_0. However,

$$\theta_{12} = \frac{1}{2}\tan^{-1}(2\sqrt{2}) = 35.26^\circ \quad \text{the tribimaximal value.}$$

Such a θ_{12} is outside the 1σ range!

However, at 3σ the data allow TBM.
For the lightest neutrino this requires 2.0×10^{-3} eV $\leq m_0 \leq 3.3\times 10^{-3}$ eV.

Complex Perturbation (i)

Introduce phases in $M_R \;\Rightarrow M_R^{flavour} = m_R \begin{pmatrix} 0 & e^{-i\phi_1} & 0 \\ e^{-i\phi_1} & 0 & 0 \\ 0 & 0 & e^{-i\phi_3} \end{pmatrix}$.

We restrict to the case $\phi_3 = 0$. Type-I see-saw then gives:

$$M'^{mass} = U^{0T} M'^{flavour} U^0 = \frac{m_D^2}{\sqrt{2}m_R} \begin{pmatrix} 0 & e^{i\phi_1} & e^{i\phi_1} \\ e^{i\phi_1} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ e^{i\phi_1} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

Work with the combination $M_{pert} = (M^{0\dagger} M' + M'^{\dagger} M^0)^{mass}$ with

$$M_{pert} = \frac{m_D^2}{\sqrt{2}m_R} \begin{pmatrix} 0 & 2m_1^{(0)} \cos\phi_1 & m^+ \cos\phi_1 - i\, m^- \sin\phi_1 \\ 2m_1^{(0)} \cos\phi_1 & \frac{2}{\sqrt{2}} m_1^{(0)} & -\frac{m^+}{\sqrt{2}} \\ m^+ \cos\phi_1 + i\, m^- \sin\phi_1 & -\frac{m^+}{\sqrt{2}} & \frac{2}{\sqrt{2}} m_3^{(0)} \end{pmatrix}.$$

Complex Perturbation (ii)

Solar sector

Solar mixing angle

$$(M^{0\dagger}M' + M'^{\dagger}M^0)^{mass}_{(2\times 2)} = \frac{\sqrt{2}m_1^{(0)}m_D^2}{m_R}\begin{pmatrix} 0 & \cos\phi_1 \\ \cos\phi_1 & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

This implies $\theta_{12} = \frac{1}{2}\tan^{-1}(2\sqrt{2}\cos\phi_1)$.

The 1σ range for $\cos\phi$ is $0.764 < \cos\phi_1 < 0.890$.

Solar mass splitting

$$\Delta m^2_{solar} = \sqrt{2}\,\sigma m_1^{(0)} m^- \sqrt{1 + 8\cos^2\phi_1}$$

where $\sigma \equiv \frac{m_D^2}{\sqrt{2}m_R m^-}$ – positive (negative) for NO (IO) – as before.

Thus solar splitting and mixing can both be satisfied with the complex perturbation.

What about θ_{13} and θ_{23}?

Complex Perturbation (iii)

θ_{13}, θ_{23}

$$|\psi_3\rangle = \begin{pmatrix} \sigma m^- z_1 \\ \frac{1}{\sqrt{2}}(1 - \frac{\sigma}{\sqrt{2}}) \\ \frac{1}{\sqrt{2}}(1 + \frac{\sigma}{\sqrt{2}}) \end{pmatrix},$$

with $z_1 \equiv \frac{\cos\phi_1}{m^-} - i\,\frac{\sin\phi_1}{m^+}$.

Now $s_{13} e^{-i\delta} = \sigma m^- z_1$. from where θ_{13} and δ can be obtained.

$$s_{13} = \sigma |m^-| \sqrt{\frac{\cos^2\phi_1}{{m^-}^2} + \frac{\sin^2\phi_1}{{m^+}^2}}$$

$$\delta = \tan^{-1}\left(\tan\phi_1 \frac{m^-}{m^+}\right),$$

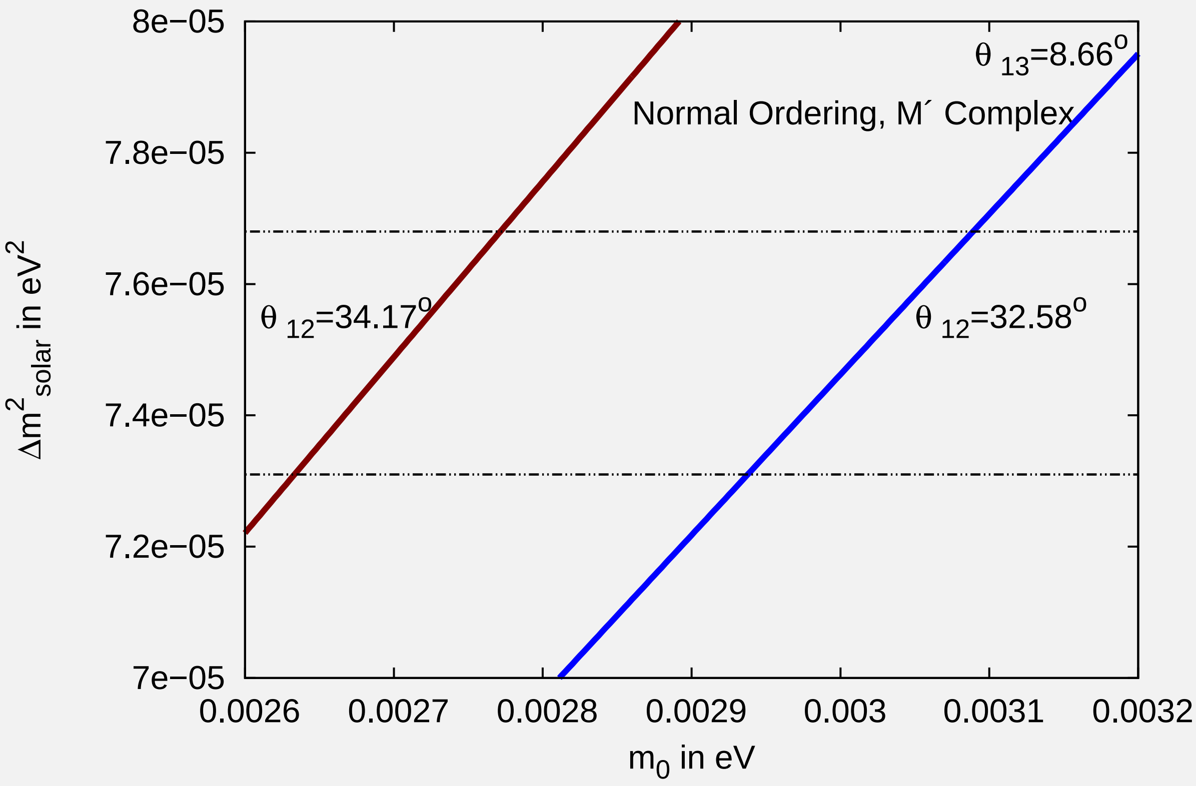
Also,

$$\tan\theta_{23} = \tan(45^\circ - \varphi') \quad \text{where} \quad \tan\varphi' = \left(\frac{s_{13}}{\sqrt{2}}\right)\left(\frac{\cos\delta}{\cos\phi_1}\right).$$

Note: Normal (Inverted) Ordering prefers first (second) octant.

Range of lightest neutrino mass

θ_{12} permits a limited range of ϕ_1.
Consistent with solar mass splitting only for a narrow range of lightest neutrino mass m_0.



The model works only for Normal ordering of masses

Central value of θ_{13} has been used.

First octant of θ_{23} is preferred.

CP-violation:
$24^o \leq \delta \leq 36^o$,
$1.4 \leq J \times 10^2 \leq 2.0$

Note the narrow range of m_0 that survives!

Conclusions

- $\theta_{13} \ll \theta_{23},\ \theta_{12}$, and $(\Delta m^2)_{solar} \ll (\Delta m^2)_{atmos}$
- Both could arise from a perturbation of a more symmetric form with $\theta_{13} = 0$ and $(\Delta m^2)_{solar} = 0$
- For a real perturbation this works only for Normal ordering
- Degeneracy in the unperturbed states removed and θ_{12} brought within 1 σ of best fits.
- With complex perturbations Inverted ordering can also be accommodated.
- Leptonic CP-violation becomes possible.
- A restrictive see-saw model with these features has been written down. Two phases are permitted. No phase (real perturbation) gives θ_{12} outside the 1σ range.
- With one phase goals are accomplished for normal ordered masses and θ_{23} is in the first octant.
- The lightest neutrino mass must lie in a limited range around 2.5×10^{-3} eV.
- Including both phases inverted ordering may be allowed but with θ_{23} within 3σ, not 1σ.

THANK YOU!!

Birthday Greetings