Two Texture Zeros and Near Maximal Atmospheric Neutrino Mixing Angle

S. Dev[*a,b], Radha Raman Gautam[†c], Lal Singh[‡a] and Manmohan Gupta[§c]

[a] *Department of Physics, Himachal Pradesh University, Shimla 171005, INDIA.*

[b] *Department of Physics, School of Sciences, HNBG Central University, Srinagar, Uttarakhand 246174, INDIA.*

[c] *Department of Physics, Centre of Advanced Study, Panjab University, Chandigarh 160014, INDIA.*

Abstract

We study the implications of a large value of the effective Majorana neutrino mass for a class of two texture zero neutrino mass matrices in the flavor basis. We find that these textures predict near maximal atmospheric neutrino mixing angle in the limit of large effective Majorana neutrino mass. We present the symmetry realization of these textures using the discrete cyclic group Z_3. It is found that the texture zeros realised in this work remain stable under renormalization group running.

1 Introduction

Two texture zeros in the effective neutrino mass matrix (M_ν) in a basis where the charged lepton mass matrix (M_l) is diagonal have been quite successful in explaining the neutrino masses and mixings [1]. Out of the 15 possible cases of two texture zeros in M_ν, only 7 are compatible with the present neutrino oscillation data. The seven allowed cases of two texture zeros in the nomenclature of Ref. [1] are listed in Table 1.

It was shown by Grimus et al. [3] that classes B_3 and B_4 of two texture zeros predict a near maximal atmospheric neutrino mixing angle when supplemented with the assumption of quasidegenerate neutrino masses and this prediction is independent of the values of solar and reactor neutrino mixing angles. In the present work, which is based on Ref. [2], we investigate the implications of a large effective Majorana neutrino mass ($|M_{ee}|$) for two texture zeros. We find that near maximal θ_{23} is predicted for classes B_1, B_2, B_3 and B_4 of two texture zeros in the limit of a large $|M_{ee}|$ and this prediction is independent of the values of solar and reactor neutrino mixing angles. The symmetry realization of these texture structures using Z_3 symmetry has been presented. It has

*dev5703@yahoo.com

†gautamrrg@gmail.com

‡lalsingh96@yahoo.com

§mmgupta@pu.ac.in

A_1	A_2	B_1	B_2
$\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$
B_3	B_4	C	-
$\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	-

Table 1: Viable two texture zero neutrino mass matrices. $\times$ denotes the non-zero elements.

been shown that the texture zeros realised in this work remain stable under renormalization group (RG) running of the effective neutrino mass matrix at one loop level.

2 The Framework

In the flavor basis the complex symmetric neutrino mass matrix for Majorana neutrinos can be diagonalized by a unitary matrix V as

$$M_\nu = V M_\nu^{diag} V^T \tag{1}$$

The matrix M_ν can be parametrized in terms of the three neutrino masses (m_1, m_2, m_3), the three neutrino mixing angles (θ_{12}, θ_{23} and θ_{13}, the solar, atmospheric and the reactor neutrino mixing angles, respectively) and the Dirac-type CP-violating phase δ. The two additional phases (α, β) appear for Majorana neutrinos. We write the matrix V as

$$V = UP \tag{2}$$

where P is a phase matrix and U is parametrized in the PDG representation. The matrix V is the neutrino mixing matrix. Two texture zeros at (p, q) and (r, s) positions lead to the following mass ratios

$$\frac{m_1}{m_2} e^{-2i\alpha} = \frac{U_{r2}U_{s2}U_{p3}U_{q3} - U_{p2}U_{q2}U_{r3}U_{s3}}{U_{p1}U_{q1}U_{r3}U_{s3} - U_{p3}U_{q3}U_{r1}U_{s1}} \tag{3}$$

$$\frac{m_1}{m_3} e^{-2i\beta} = \frac{U_{r3}U_{s3}U_{p2}U_{q2} - U_{p3}U_{q3}U_{r2}U_{s2}}{U_{p1}U_{q1}U_{r2}U_{s2} - U_{p2}U_{q2}U_{r1}U_{s1}} e^{2i\delta} \ . \tag{4}$$

The magnitudes of the two mass ratios in Eqs. (3) and (4), are denoted by

$$\eta = \left| \frac{m_1}{m_2} e^{-2i\alpha} \right|, \qquad \rho = \left| \frac{m_1}{m_3} e^{-2i\beta} \right|. \tag{5}$$

The two mass ratios can be further used to obtain two values of m_1 viz.

$$m_1 = \eta \sqrt{\frac{\Delta m_{21}^2}{1 - \eta^2}} \ , \quad m_1 = \rho \sqrt{\frac{\Delta m_{21}^2 + |\Delta m_{23}^2|}{1 - \rho^2}} \tag{6}$$

where $(\Delta m_{ij}^2 \equiv m_i^2 - m_j^2)$. The above two values of m_1 contain the constraints of two texture zeros in M_ν through the two mass ratios η and ρ. The simultaneous existence of two texture zeros in M_ν requires these two values of m_1 to be equal. For complete details of the formalism see Ref. [2]

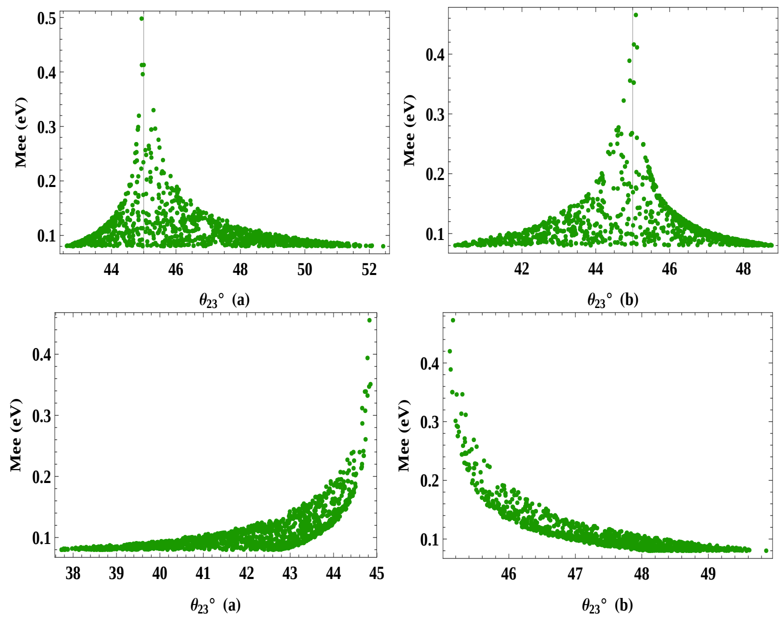

Figure 1: Correlation plots for classes B_1(NH)(a), B_1(IH)(b), B_3(NH)(c) and B_3(IH)(d). Here the mixing angles are varied between 0° and 90° [2].

3 Numerical Analysis

The effective Majorana mass $|M_{ee}|$ which determines the rate of neutrinoless double beta (NDB) decay is given by

$$|M_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}|. \tag{7}$$

In the present work, we take the upper limit on $|M_{ee}|$ to be 0.5 eV [5]. For the numerical analysis we have used the recent global fits on neutrino oscillation parameters [6]. In the numerical analysis, first we use the experimental input of the two mass squared differences $(\Delta m_{21}^2, \Delta m_{23}^2)$ along with the constraints of two texture zeros and large $|M_{ee}|$ to obtain predictions for mixing angles. We vary the two mass squared differences randomly within the 3σ allowed ranges but keep the neutrino mixing angles free and vary them between 0° and 90°. The Dirac phase is varied from 0° to 360° and the constraint of a large $|M_{ee}| > 0.08$ eV is imposed. It is found that all these classes predict a near maximal atmospheric neutrino mixing angle while the other two mixing angles remain unconstrained. The atmospheric neutrino mixing angle remains near maximal irrespective of the values of solar and reactor neutrino mixing angles [see Fig. (1)]. Grimus et al. [3] have shown that only for classes B_3 and B_4 a near maximal θ_{23} is predicted in the limit of a quasi degenerate (QD) spectrum. In comparison, our assumption of a large $|M_{ee}|$ leads to a near maximal θ_{23} for all the four classes (B_1, B_2, B_3 and B_4). In the second step, we also take into account the experimental input of mixing angles along with the experimental input of the two mass squared differences [see Fig. (2)].

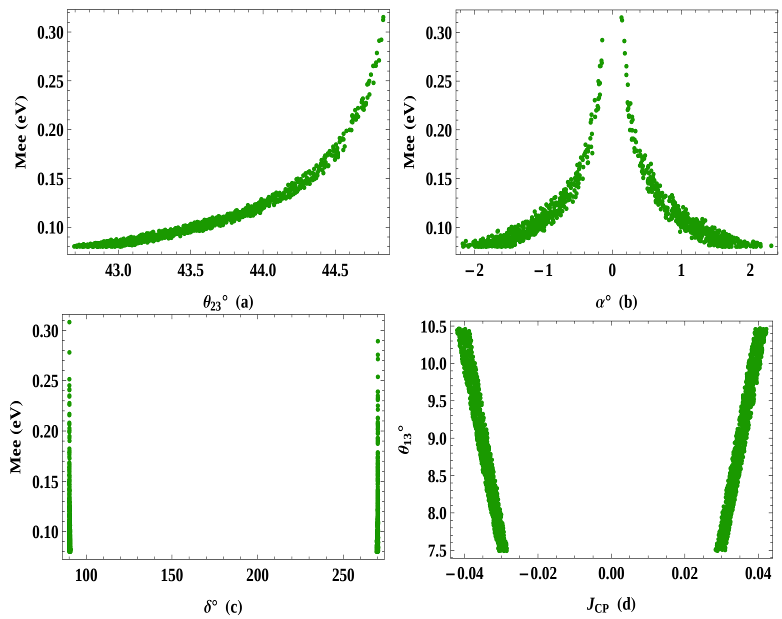

Figure 2: Correlation plots for class B_1(NH)(a, b) and B_3(IH)(c, d) [2].

Class	$D_{eL},\ D_{\mu L},\ D_{\tau L}$	$e_R,\ \mu_R,\ \tau_R$	$\nu_{eR},\ \nu_{\mu R},\ \nu_{\tau R}$	ϕ	$\triangle$	χ
B_1	$\omega,\ \omega^2,\ 1$	$\omega,\ \omega^2,\ 1$	$\omega,\ \omega^2,\ 1$	1	ω	-
B_2	$\omega^2,\ 1,\ \omega$	$\omega^2,\ 1,\ \omega$	$\omega^2,\ 1,\ \omega$	1	ω^2	-
B_3	$1,\ \omega^2,\ \omega$	$1,\ \omega^2,\ \omega$	$\omega,\ 1,\ 1$	1	1	ω^2
B_4	$1,\ \omega,\ \omega^2$	$1,\ \omega,\ \omega^2$	$\omega,\ 1,\ 1$	1	1	ω^2

Table 2: The transformation properties of lepton and scalar fields under Z_3 for classes B_1, B_2, B_3 and B_4. Here, $\omega = e^{i2\pi/3}$, D_{lL} $(l = e, \mu, \tau)$ denotes $SU(2)_L$ doublets and l_R, ν_{lR} denote the right-handed $SU(2)_L$ singlet charged lepton and neutrino fields, respectively. ϕ, $\triangle$ and χ denote $SU(2)_L$ doublet, triplet and singlet scalars, respectively [2].

4 Symmetry Realization

General guidelines for the symmetry realization of texture zeros in both the quark and the lepton mass matrices have been given in Ref. [7] which outlines procedures for enforcing texture zeros at any place in the fermion mass matrices by imposing discrete Abelian family symmetries. To obtain the desired texture structures for class B, we use the framework of type-I+II seesaw mechanism [8, 9]. For the symmetry realization, we use the discrete cyclic group Z_3. In Table 2, we have summarized the transformation properties of lepton and scalar fields under the action of Z_3 for classes B_1, B_2, B_3 and B_4.

All the above classes of texture zeros are realized at the seesaw scale which poses the question whether the texture zeros realized in this work survive when the RG evolution of M_ν from the seesaw to the electroweak scale is taken into account. For all the Yukawa coupling matrices realized in this work, the hermitian products $Y_k Y_k^\dagger$ $(k = l, \nu, \triangle)$ which are relevant for the RG evolution of M_ν come out to be diagonal so that RG corrections are multiplicative on the effective neutrino mass matrix elements leaving zero elements intact. Thus, although the values of neutrino masses and neutrino mixing parameters change due to RG corrections while running down from the seesaw scale to the electroweak scale, the correlations induced between neutrino masses and mixing parameters by texture zeros remain unchanged due to the stability of texture zeros atleast at one loop level.

5 Summary

We have studied the implications of large effective Majorana neutrino mass for a class of two texture zero neutrino mass matrices. We found that classes B_1, B_2, B_3 and B_4 all predict near maximal atmospheric neutrino mixing angle when supplemented with the assumption of large effective Majorana neutrino mass. The near maximality of the atmospheric neutrino mixing angle is independent of the values of the solar and reactor neutrino mixing angles. Furthermore, we have shown how one can obtain such texture structures in the context of type-I+II seesaw using the discrete Abelian group Z_3. The texture zeros realised in this work remain stable under renormalization group running. The assumption of large $|M_{ee}|$ is motivated by the fact that there are a number of forthcoming and presently ongoing experiments searching for neutrinoless double beta decay.

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References

[1] Paul H. Frampton, Sheldon L. Glashow and Danny Marfatia, *Phys. Lett.* **B 536**, 79 (2002), hep-ph/0201008.

[2] S. Dev, R. R. Gautam, L. Singh and M. M. Gupta *Phys. Rev.* **D 90**, 013021 (2014), arXiv:1405.0566 [hep-ph].

[3] W. Grimus, P. O. Ludl, *Phys. Lett.* **B 700**, 356-361 (2011), arXiv:1104.4340 [hep-ph].

[4] E. I. Lashin and N. Chamoun, *Phys. Rev.* **D 78**, 073002 (2008), arXiv:0708.2423 [hep-ph].

[5] W. Rodejohann, *Int. J. Mod. Phys.* **E, 20**, 1833 (2011), arXiv:1106.1334 [hep-ph].

[6] D. V. Forero, M. Tortola and J. W. F. Valle , *Phys. Rev.* **D 86**, 073012 (2012), arXiv:1205.4018 [hep-ph].

[7] W. Grimus, A. S. Joshipura, L. Lavoura and M. Tanimoto, *Eur. Phys. J.* **C 36**, 227 (2004), hep-ph/0405016.

[8] P. Minkowski, *Phys. Lett.* **B 67**, 421 (1977).

[9] W. Konetschny and W. Kummer, *Phys. Lett.* **B 70**, 433 (1977); T. P. Cheng and L. F. Li, *Phys. Rev.* **D 22**, 2860 (1980).

[10] W. Rodejohann, M. A. Schmidt, *Phys. Atom. Nucl.* **69**, 1833 - 1841 (2006), hep-ph/0507300.